MEMS Inertial Sensors Modeling Using Power Spectral Density Method

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Abstract

Micro-electromechanical systems (MEMS) such as accelerometers and gyroscopes - which are also known as inertial sensors - are suitable for the inertial navigation system (INS) of various applications due to their low price, small dimensions, and lightweight. Some of these sensors are capable of delivering extremely precise acceleration and angular

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velocity data. There is, however, no inertial sensor that can produce the flawless acceleration and angular velocity data. There will always be flaws in the inertial data it returns, no matter how good it is. These flaws are caused by a variety of faults, each of which has a different impact on the data. There are two main parts of faults in inertial sensors, deterministic and the stochastic. The deterministic part can be managed and calculated by calibration and then removed from the raw data. On the other hand, the stochastic part contains random errors (noises) in the system cannot be removed from measurements, hence the stochastic processes model should be introduced. In this paper, the power spectral density (PSD) will be used in modeling the stochastic errors of the inertial sensors. A characteristic curve is created by applying a simple operation on the entire length of data, and its inspection gives a systematic characterization of the random errors contained in the inertial-sensor output data.

Keywords: *Accelerometer, gyroscope, power spectral density, stochastic errors.*

I. Introduction

Micromachining and (MEMS) technologies can be used to create sophisticated structures, devices and systems on the scale of micrometers. Micromachining techniques were originally taken directly from the integrated circuit (IC) technology, but currently, a variety of MEMS-specific micromachining technologies are being developed. A wide range of transduction mechanisms can be used in MEMS to convert real-world signals from one form of energy to another, enabling a wide range of micro-sensors, micro-actuators, and microsystems.) [1].

Advances in Micro-Electromechanical Systems (MEMS) technology, combined with the miniaturization of electronics, have enabled the introduction of light-weight, low-cost, and low-power chip-based inertial sensors for measuring angular velocity and acceleration as an alternative to more expensive traditional inertial sensors.

MEMS-based inertial sensors have a variety of applications in low-cost navigation and control systems. The significant errors that accompany the corresponding measurements are a common drawback of these sensors. These errors have both deterministic and stochastic parts. The deterministic part includes constant biases, scale factors, axis non-orthogonality, axis misalignment, and other factors that are removed from row measurements using the appropriate calibration techniques. The stochastic part contains random errors that cannot be removed from measurements and should be modeled as stochastic processes. [1].

A typical inertial navigation system employs a combination of roll, pitch, and azimuth gyroscopes to stabilize the x, y, and z accelerometers, which are then used to solve a large set of differential equations to convert these readings into estimates of velocities, position, and attitude, beginning with a known initial position of latitude and longitude.

Inertial navigation systems (INS) can provide high-accuracy position, velocity, and attitude data in short periods. Their accuracy, however, degrades rapidly over time. The requirements for an accurate estimation of navigation information necessitate the modeling of the sensor’s error components. Several variance techniques have been developed for stochastic modeling of inertial sensor error. They are basically very similar and primarily differ in that various signal processing. Power spectral density (PSD) will be used in modeling the stochastic errors of the inertial sensors. A characteristic curve is obtained by performing a simple operation on the entire length of data, and its inspection provides a systematic characterization of various random errors contained in the inertial-sensor output data.

II. Power Spectral Density (PSD)

In stochastic modeling, there may be no direct access to the input. A model is proposed that, when excited by white noise, has the same output characteristics as the unit under test. Because such models are not generally unique, certain canonical forms are chosen.

A stochastic model is a tool used to estimate the probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. The random variation is typically based on fluctuations observed in historical data for a given time period using standard time-series techniques. The potential outcome distributions are derived from many simulations (stochastic projections) that reflect the random variation in the input (s).

The noise in measurements x(t) arising from the accelerometer or another inertial instrument testing can usefully be characterized by modeling x(t) as a stochastic process and applying power spectral density or other noise estimation techniques.

A stochastic process consists of a family of (possibly vector-valued) random variables that are taken from a probability space Ω and indexed by a parameter t. A stochastic process is written as x(t, ω) or simply x(t), where ω is a point in Ω and t is the time variable [2].

The application of white noise and the construction of the transfer function in this manner is important in stochastic modeling. Instead of obtaining the cross PSD between input and output, the transfer function can be estimated solely from the output power spectrum. Thus, for a linear time-invariant system, characterizing the unknown model is possible by knowing only the output and assuming white noise inputs.

Many of the methods are very similar to those used in dynamic modeling, with the exception that the input is unobservable [3].

III. Methodology

The frequency domain approach of estimating transfer functions using the power spectral density is simple but difficult for non-system analysts to understand. The Power Spectral Density (PSD) is the most commonly used representation method of the spectral decomposition of a time series. It is an effective tool for analyzing or characterizing data as well as stochastic modeling. The PSD, or spectrum analysis, is also more suitable than other methods for analyzing periodic or non-periodic signals[3].

The PSD of a stationary stochastic c process is defined as, the Fourier transform of its autocorrelation function K(τ) [3]:

$$S(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega\tau} K(\tau) d\tau \qquad (1)$$

and

$$K(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{j\omega\tau} S(\omega) d\omega \qquad (2)$$

Because the autocorrelation function of a stationary stochastic process is even, the PSD is also even, so the one-sided PSD equals [4]:

$$S^1(\omega) = 2 \int_0^{\infty} e^{-j\omega\tau} K(\tau) d\tau \qquad (3)$$

Data is typically collected at discrete intervals using a digital computer. Consider N samples of the sensor output with sample time Δt. Thus, the length of the time ensembles is T=N×Δ. In the following computations, the one-side of PSD estimate is given by [5]:

$$S^1(f) = \frac{1}{T} |X(f)|^2 \qquad (4)$$

where $X(f)$ is the Fourier transform of the measured time series $x(t)$. The discrete Fourier transform approximates the continuous Fourier transform at discrete frequencies f_j by

$$X(f_j) \cong X_j \Delta t \quad (5)$$

with $$f_j = \frac{j}{N.\Delta t} = \frac{j}{T} Hz \quad (6)$$

Thus, the estimate from a finite span of sampled data of the one-sided PSD at frequency f_j is [5]

$$S^1(f) = \frac{\Delta t^2}{T} \left|X_j\right|^2, \qquad j = 1, 2, \dots, \left[\frac{N}{2}\right] \quad (7)$$

The one-sided PSD is frequently used in data analysis where it is convenient to consider frequency f to be positive, whereas the two-sided PSD is more convenient for mathematical proofs.

A. Angle (velocity) Random Walk

The gyro angle (or accelerometer velocity) random walk can be influenced by a high frequency noise term with a correlation time much shorter than the sample time. These noise terms are all characterized by a white-noise spectrum on the gyro (or accelerometer) output rate. Fig. 1 represents the PSD of an accelerometer approximated with the zero slopes characteristic. The associated rate noise PSD is represented by [6]:

$$S_\Omega(f) = N^2 \quad (8)$$

where N is the angle (velocity) random walk coefficient.

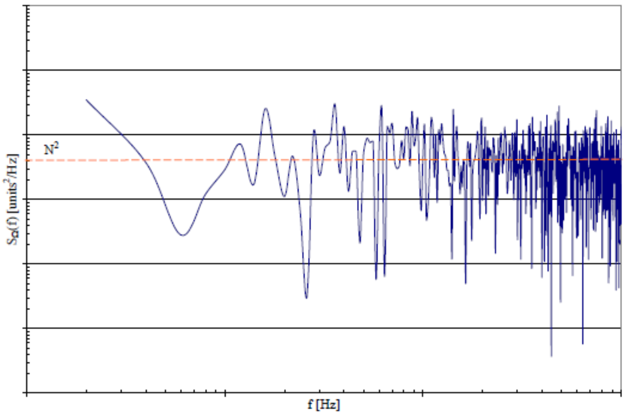


Figure 1. White noise spectrum of the accelerometer rate output

The value of velocity random walk can determine by using the relation between the PSD and the velocity random walk coefficient N is [3], [7]:

$$N\left((m/s)/\sqrt{h}\right) = \frac{1}{60}\sqrt{PSD\left[\frac{\left((m/s)/h\right)^2}{Hz}\right]} \qquad (9)$$

B. Quantization Noise

Quantization noise is one of the types of error introduced into an analog signal that results from encoding it in digital form. Quantization noise is caused by the small differences between the actual amplitudes of the points being sampled and the bit resolution of the analog-to-digital converter.

The angle PSD for a gyro output, for example, is given in [8] as:

$$S_\theta(f) = \tau Q_z^2\left(\frac{\sin^2(\pi f\tau)}{(\pi f\tau)^2}\right) \approx T_s Q_z^2 \qquad f < \frac{1}{2T_s} \qquad (10)$$

Where Q_z is the quantization-noise coefficient and T_S is the sample interval.

The theoretical limit for Q_z is equal to $S/\sqrt{12}$, where S is the gyro scaling coefficient for the tests with fixed and uniform sampling times. The gyro rate PSD, on the other hand, is related to the angle PSD through the following relationship [8]:

$$S_\Omega(2\pi f) = (2\pi f)^2 S_\theta(2\pi f) \qquad (11)$$

And is

$$S_\Omega(f) = \frac{4Q_Z^2}{T_S}\sin^2(\pi f T_S) \approx (2\pi f)^2\, T_S Q_Z^2,\ \ f < \frac{1}{2T_S} \qquad (12)$$

Fig. 2 represents the PSD of an accelerometer approximated with the 2 slope characteristic.

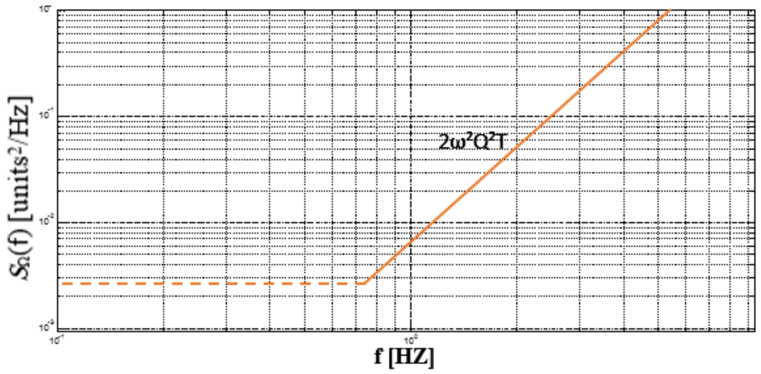


Figure 2. the Quantization noise of the accelerometer rate output [3]

C. Bias instability

Bias Instability is also referred to as "flicker noise." This is an example of low frequency bias fluctuations in measured rate data. The source of this noise is the electronics or other components susceptible to random flickering [9]. Because of its low frequency, it emerges as bias fluctuations in the data. The PSD associated with this noise has the following rate: [8]:

$$S_\Omega(f) = \begin{cases} \left(\frac{B^2}{2\pi}\right)\frac{1}{f} & f \le f_0 \\ 0 & f > f_0 \end{cases} \qquad (13)$$

Where *B* is the bias instability coefficient and f_0 is the cutoff frequency.

Fig. 3 represents the PSD of an accelerometer approximated with the -1 slope characteristic.

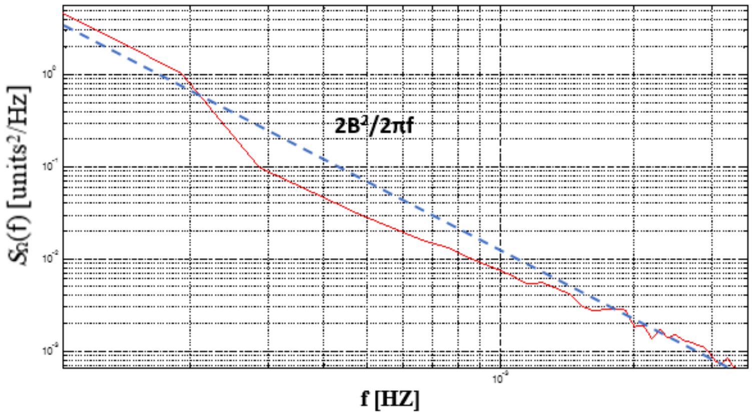


Figure 3. Bias Instability or flicker noise of the accelerometer rate output [3]

Bias instability parameter is obtained by fitting a straight line with a slope of -1 until it meets the vertical line of f = 1 Hz, using the relation [3], [10]:

$$B(m/s^2) = 2 \times \sqrt{PSD\left[\frac{(m/s^2)^2}{Hz}\right]} \qquad (6.2)$$

D. Rate Random Walk

This is a random process of uncertain origin, possibly a limiting case of an exponentially correlated noise with a very long correlation time. The rate PSD associated with this noise is [3]:

$$S_\Omega(f) = \left(\frac{K}{2\pi}\right)^2 \frac{1}{f^2} \qquad (14)$$

where *K* is the rate random walk coefficient.

Fig. 4 represents the PSD of an accelerometer approximated with the -2 slope characteristic

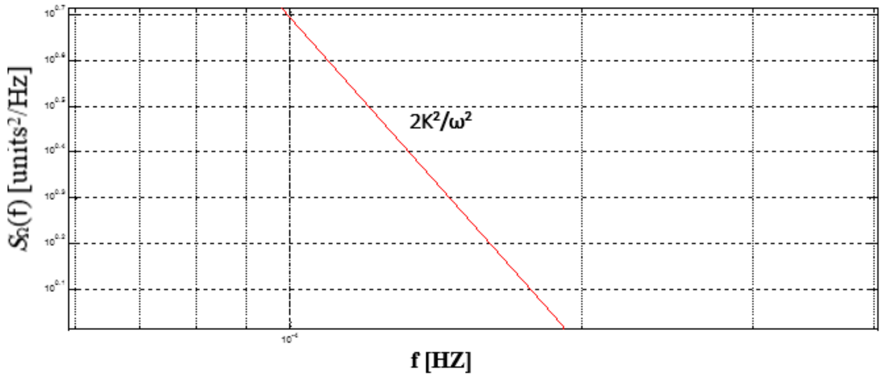
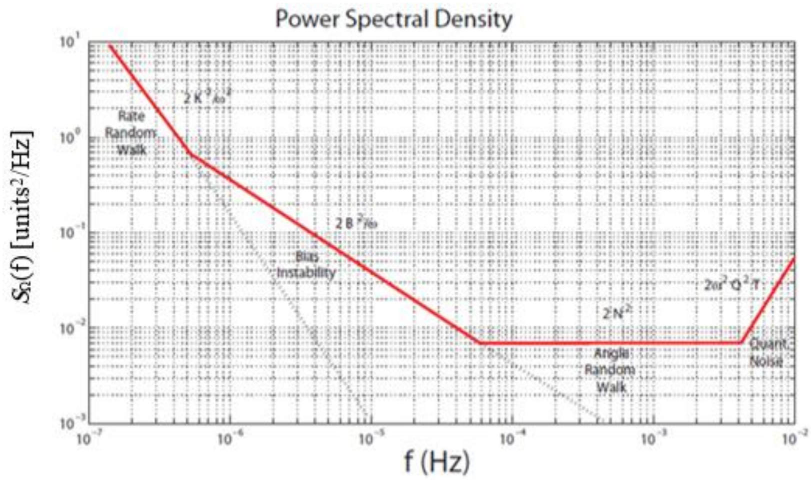


Figure 4. Rate Random Walk noise of the accelerometer rate output [3]

Types of noise on the measurement are different which can be represented in the PSD by straight lines with varying slopes. In Fig. 5 the typical characteristic sloped is shown. In real data, gradual transitions would exist between the different PSD slopes, rather than sharp transitions in the Fig. 5.

Figure 5:



Hypothetical gyro in single-sided PSD form [3]

IV. Experiment and Test

The proposed Power Spectral Density (PSD) method was applied to the real data collected from the gyroscope and the accelerometer of Sparkfun 9DOF Razor inertial measurement unit (IMU) sensor. This IMU sensor is shown in the Fig. 6 which includes InvenSense ITG-3200 triple-axis digital output gyroscope, Analog Devices ADXL345 triple-axis accelerometer and HMC5883L triple-axis digital magnetometer. The ADXL345 is a small, thin, ultralow power, 3-axis accelerometer with high resolution (13-bit) measurement at up to ±16 g. Digital output data is formatted as 16-bit twos complement and is accessible through either a SPI (3- or 4-wire) or I2C digital interface [12]. The ADXL345 is supplied in a small, thin, 3 mm × 5 mm × 1 mm.

Fig. 6: Sparkfun 9DOF Razor IMU

The ITG-3200 is the world's first single-chip, digital-output, 3-axis MEMS gyro IC. Features three 16-bit analog-to-digital converters (ADCs) for digitizing the gyro outputs, a user-selectable internal low-pass filter bandwidth, and a Fast-Mode I2C (400kHz) interface [18]. The ITG-3200 uses InvenSense's proprietary MEMS technology with vertically driven vibrating masses to produce a functionally complete, low-cost motion sensor. All required conditioning electronics are integrated into a single chip measuring 4 x 4 x 0.9 mm [11].

The IMU was placed on a flat surface stationary and as horizontal as possible, it was placed on concrete, not a wooden table for 8 hours without external environmental disturbance to the system, at stable room temperature. The output of the IMU were connected via the Arduino Uno board to the MATLAB environment, Arduino was used in this project as an interface between IMU and the computer. Accelerometer and gyroscope outputs are recorded to file at a sampling rate of 50Hz will result in 50Hz × 8 hours = 1440000 samples. The experiment layout and the equipment used in this experiment are shown in Fig. 7.

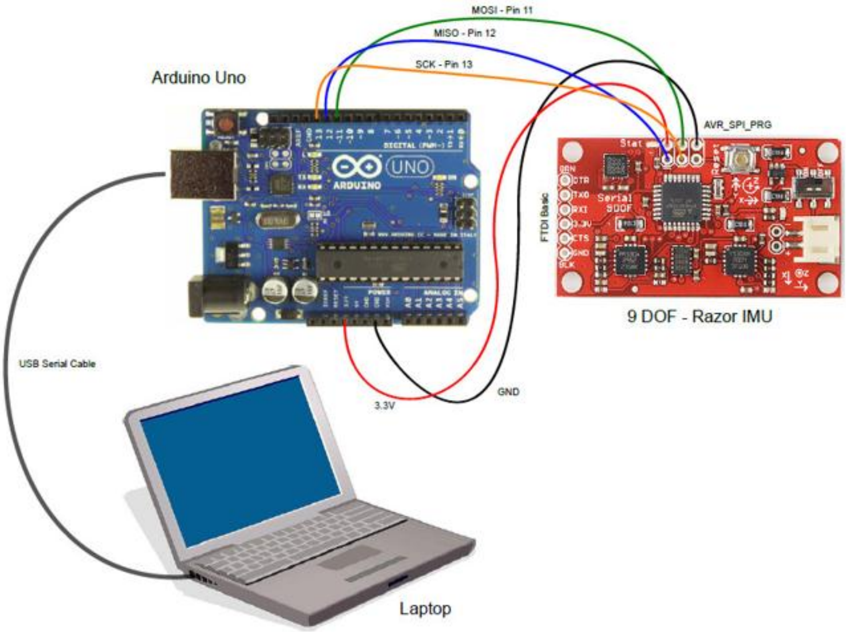


Fig. 7: Experiment Setup to collect the IMU data

V. Results Analysis

The power spectral density method in the MATLAB environment is applied to the acquired data for the accelerometer and gyroscope, each axis must be evaluated separately assuming that they are independent of each other. So there are different error coefficients for each axis of accelerometer and gyroscope. By applying the proposed method to the whole data set, the results for the PSD of the gyroscope and accelerometer versus frequency are shown in Fig. 8 and 9 respectively.

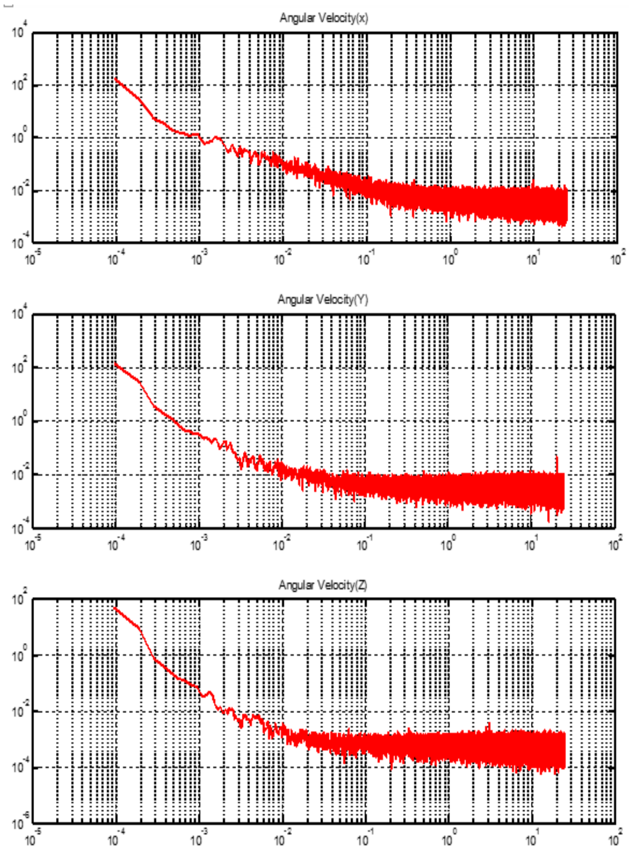


Fig. 8: PSD plot of triple-axis ITG-3200 Gyroscope

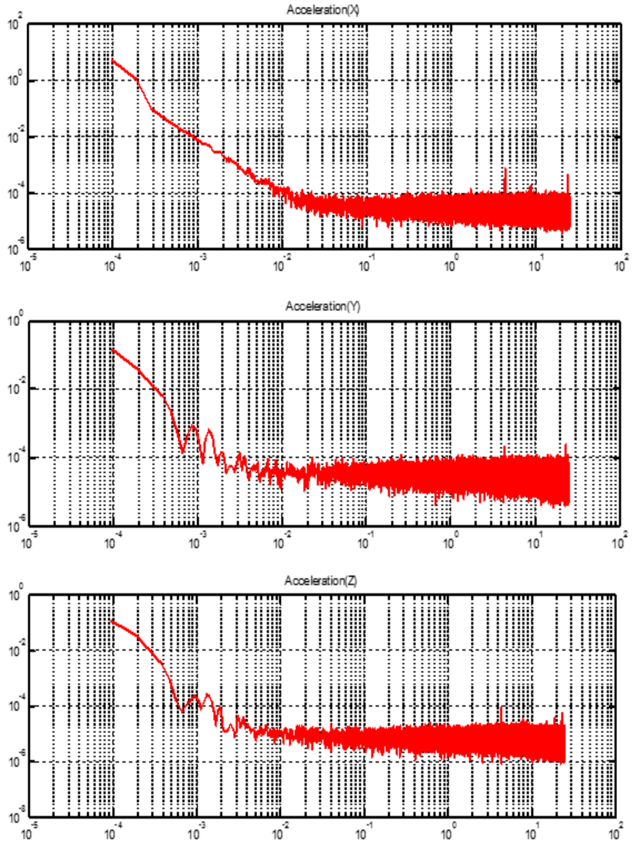


Fig. 9: PSD plot of triple-axis ADXL345 Accelerometer

From the collected static data, we can get the PSD plot. The PSD of the accelerometer along Y-axis is shown in Fig. 10. There are three types of noise: Acceleration random walk (*K*), bias instability (*B*), and velocity random walk (*N*).

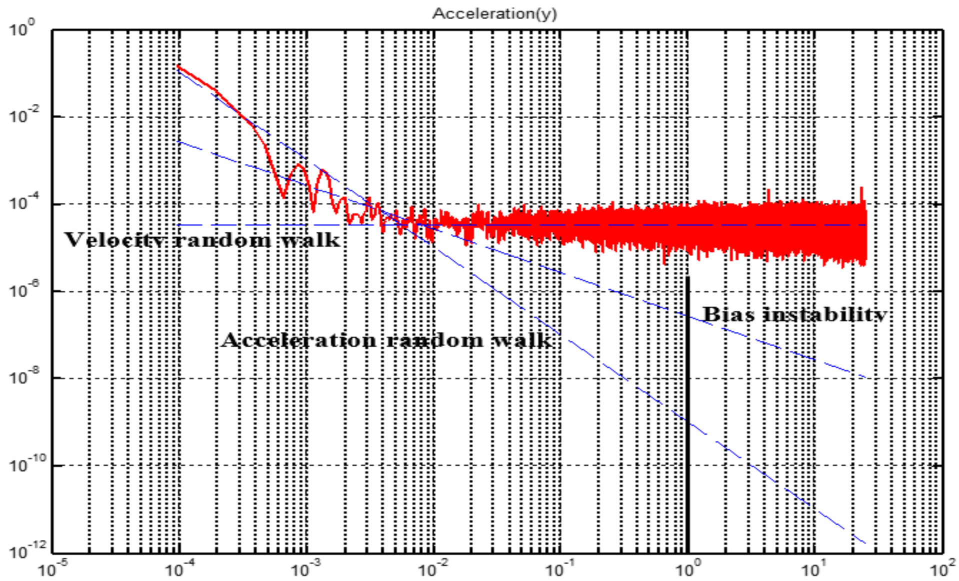


Figure 10: PSD of the Y accelerometer with slopes

The values for each noise parameter (*B*,*N*,*K*) were extracted by drawing straight lines for each frequency band influenced by the noise. The interception of each line with a specific point was considered. For instance, the PSD curve for the Y-axis accelerometer is plotted in Fig. 10, it also includes straight dotted lines for each noise, *N*, *B* and *K*, with their respective slopes, 0, -1, -2. The acceleration random walk (*K*) is present in the low-frequency components between 10-4Hz and 5.5×10-3 Hz. This parameter is obtained by fitting a straight line with a slope of -2, starting from 10-4 Hz, until it meets the vertical line of f = 1 Hz.

Tables 1 and 2 Summarize the values of different errors that affect the inertial sensors used in the test, using PSD method of the accelerometer and gyroscope respectively.

Table 1: Accelerometer parameters specification using PSD method

Parameter	Determined value	Unit
Velocity Random Walk	$N(a_x)$=0.0530	m/s/(h)$^{0.5}$
	$N(a_y)$=0.0362	
	$N(a_z)$=0.0778	
Bias instability	$B(a_x)$=2.6771×10^{-4}	m/s^2
	$B(a_y)$=1.3409×10^{-4}	
	$B(a_z)$=2.0107×10^{-4}	
Acceleration Random Walk	$K(a_x)$=5.9497×10^{-5}	m/s/s^2
	$K(a_y)$=2.9710×10^{-5}	
	$K(a_z)$=5.1460×10^{-5}	

Table 2: Gyroscope parameters specification using PSD method

Parameter	Determined value	Unit
Angle Random Walk	$N(\omega_x)$=0.5186	Deg/(h)$^{0.5}$
	$N(\omega_y)$=0.4715	
	$N(\omega_z)$=0.4734	
Bias instability	$B(\omega_x)$=0.0086	Deg/s
	$B(\omega_y)$=0.0032	
	$B(\omega_z)$=0.0031	
Rate Random Walk	$K(\omega_x)$=4.1698×10^{-4}	Deg/s^2
	$K(\omega_y)$=2.4054×10^{-4}	
	$K(\omega_z)$= 2.6191×10^{-4}	

VI. Conclusions

From the experiment results, we can conclude that there isn't a slope 2 on the plot of Power Spectral Density. Consequently, the quantization errors are much less than other errors and can be ignored, this because the sensor's output is 16-bit and 13bit digital data measured using on-chip ADCs, as the number of bits increases the quantization error decreases.

The results clearly indicate that the angle (velocity) random walk is the dominant error term in the high frequency, whereas the bias instability and rate random walk are the dominant errors in the small frequency.

This work clearly shows that the Allan variance analysis is a powerful technique to investigate the sensor error behaviors on different time scales.

The Power Spectral Density method is helpful in IMU analysis and modeling. This work clearly shows that the Power Spectral Density analysis is a powerful technique to investigate the sensor error behaviors on different time scales, this analysis is an effective method for error modeling and parameter estimation, these error coefficients must be identified reliably as far as possible. Because these coefficients form the error covariance matrix (process noise), that can be used in the operation navigation systems.

VII. Reference

[1]. J. W. Judy, "Micro electromechanical systems (MEMS): fabrication, design and applications", Electrical Engineering Department, University of California, Los Angeles, 2001.

[2]. IEEE Std 1293-1998, IEEE Standard Specification Format Guide and Test Procedure for Linear,Single- Axis, Nongyroscopic Accelerometers, 25 Sept. 1998, ISBN 0-7381-1430-8.

[3]. IEEE Std 952-1997 (R2008), "Standard Specification Format Guide and Test Procedure for Single Axis Interferometric Fiber Optic", 16 Sept. 1997, ISBN 1-55937-961-8.

[4]. A. Papoulis, "Probability, Random Variables and Stochastic Processes", 1991 MacGraw-Hill.

[5]. H. Hou, "Modeling Inertial Sensors Errors Using Allan Variance," PhD Thesis, Department of Geomatics Engineering, Univ. of Calgary, Calgary, Alberta, Canada. 2004. [Online]. Available: http://www.url.com

[6]. W Stockwell, Angle Random Walk, http://www.xbow.com

[7]. Oliver J. Woodman, "An introduction to inertial navigation", University of Cambridge 2007

[8]. N. El-Sheimy, H. Hou and X. Niu, "Analysis and Modeling of Inertial Sensors Using Allan Variance", IEEE Transactions On Instrumentation And Measurement, Vol. 57, No. 1, January 2008.

[9]. D. W. Allan, "Statistics of atomic frequency standards", Proceedings of the IEEE 54, 2 (Feb. 1966), 221–230.

[10]. P. Petkov, T. Slavov, "Stochastic Modeling of MEMS Inertial Sensors", Bulgarian Academy Of Sciences, Cybernetics And Information Technologies. Volume 10, No 2. Sofia 2010

[11]. ITG-3200 Product Specification Revision 1.7, 2011. [Online]. Available: https://www.glynstore.com/content/docs/invensense/PS-ITG-3200A.pdf