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Stability and Control of Non-Linear Dynamical System Subjected to Multi External Force with Velocity Feedback

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Abstract:

The response of a dynamical non-linear system of two-degree-of freedom, subjected to multi excitations forces is investigated. Analysis of the amplitude and phase plane are obtained. The multiple scale analyses of various 1:2 internal resonance conditions and simultaneous resonance case $\Omega = \omega_1,\ \omega_1 = 2\omega_2$ are considered. The method of multiple time scale (MTS) is applied to solve the non-linear differential equations describing the system up to second order approximation. All possible resonance cases at this approximation are obtained and studied numerically to determine the worst case. The effects of different parameters are studied. The frequency response equations are solved numerically. These vibrations were controlled using the damper $R_1 = -\varepsilon G_1 \dot{x}^3,\ R_2 = -\varepsilon G_2 \dot{y}^3$.

الملخص:

تم التحقيق في استجابة نظام ديناميكي غير خطي من الدرجة الثانية والذي يتعرض لقوى خارجية متعددة. يتم الحصول على السعة تحليليا. وقد تم دراسة التحليلات متعددة النطاق لمختلف شروط الرنين الداخلي والرنين المختلط $\Omega = \omega_1,\ \omega_1 = 2\omega_2$ بنسبة 1:2. يتم تطبيق طريقة المقياس (MTS) لحل المعادلات التفاضلية غير الخطية التي تصف النظام حتى التقريب من الدرجة الثانية. يتم الحصول على جميع حالات الرنين الممكنة عند هذا التقريب ودراستها عدديا لتحديد الحالة الاسوأ. تمت دراسة تأثيرات المعاملات المختلفة. تحل معادلات استجابة التردد عدديا. تم التحكم في الاهتزازات باستخدام المخمد $R_1 = -\varepsilon G_1 \dot{x}^3,\ R_2 = -\varepsilon G_2 \dot{y}^3$.

Keywords: Vibration control; Resonance Cases; Multiple time Scale; Frequency Response Curves; Stability.

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1. Introduction:

Chaos is one of the most exciting topics in the field of physical sciences. Researchers from various fields devoted too much effort in the analysis of chaotic behavior as well as the control of both vibrations and chaos for various vibrating systems. Many ideas and approaches for controlling chaos have been proposed in the past twenty years [1-5]. Zhang [6] analyzed the global bifurcation and chaotic dynamics of a parametrically excited, simply supported rectangular thin plate. The method of multiple scales is used to obtain the averaged equations in the presence of 1:1 internal resonance and primary parametric resonance. Zhang et al. [7] investigated the local and global bifurcations of a parametrically and externally excited simply supported rectangular thin plate subjected to transversal and in plane excitation simultaneously.

Belhaq et al. [8] investigated the control of chaos of one-degree-of-freedom system with both quadratic and cubic nonlinearities subjected to combined parametric and external excitations. Glabisz [9] studied the stability of one-degree-of-freedom system under velocity and acceleration dependent non-conservative forces. Eissa and Amer [10] controlled the vibration of a second order system simulating the first mode of a cantilever beam subjected to primary and sub-harmonic resonance using cubic velocity feedback. El-Bassiouny [11] made an investigation on the control of the vibration of the crankshaft in internal combustion engines subjected to both external and parametric excitations via an elastomeric absorber having both quadratic and cubic stiffness nonlinearities. Sayed and Hamed [12] studied the response of a two degree-of freedom system with quadratic coupling under parametric and harmonic excitations. The method of multiple scale perturbation technique is applied to solve the non-linear differential equations and obtain approximate solutions up to and including the second-order approximations. Abe et al. [13] investigated the non-linear responses of clamped laminated shallow shells with 1:1 internal resonance between two antisymmetric modes the frequency-response curves were obtained by the shooting method. Eissa and Sayed [14, 15] made a comparison between the active and the passive vibration control of a simple pendulum described by a second order nonlinear differential equation having both quadratic and cubic nonlinearities. they controlled the system applying either nonlinear absorber (passive control) or negative velocity feedback or its square or cubic value (active control). Yaman et al. [16] studied the problem of suppressing the vibrations of a nonlinear system with a cantilever beam of varying orientation subjected to parametric and direct excitation. They applied the cubic velocity feedback to the system to reduce the amplitudes of the system.

Chang et al. [17] investigated the bifurcations and chaos of a rectangular thin plate with 1:1 internal resonance. Tian et al. [18] studied the dynamics of a shallow arch subjected to harmonic excitation in the presence of both external and 1:1 internal resonance. Anlas and Elbeyli [19] studied the non-linear response of rectangular and square metallic plates subject to transverse harmonic excitations. Frequency response curves are presented for both square and rectangular plates for primary resonance of either mode in the presence of a 1:1 internal resonance. Ye et al. [20, 21] dealt with the non-linear dynamic behaviors of a parametrically excited, simply supported, symmetric cross-ply composite laminated rectangular thin plate and a simply supported antisymmetric cross-ply composite laminated rectangular thin plate under parametric excitation. The study is focused on the case of 1:1 internal resonance and primary parametric resonance. Guo et al. [22] dealt with the non-linear dynamics of a four-edge simply supported angle-ply composite laminated rectangular thin plate excited by both the in-plane and transverse loads. Amer and Sayed [23], studied the response of one-degree-of freedom, non-linear system under multi-parametric and external excitation forces simulating the vibration of the cantilever beam.

In the present paper, the non-linear vibrations and stability subjected to the transverse and in plane excitations simultaneously are investigated. The method of multiple time scale is applied to obtain the second-order uniform asymptotic solutions for the case of simultaneous primary in the presence of 1:2 internal resonances. All possible resonance cases are extracted and investigated at this approximation order. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system. Such cases should be avoided as working conditions for the system. The stability of the system is investigated with frequency response curves and phase-plane method. Some recommendations regarding the different parameters of the system are reported.

2. Mathematical Analysis:

$$\ddot{x} + 2\varepsilon\zeta_1\omega_1\dot{x} + \omega_1^2 x + \varepsilon\alpha_1 x^3 = \varepsilon\beta y^2 + \varepsilon f_1\cos(\Omega t) + \varepsilon f_2\cos(2\Omega t) + R_1 \quad (1)$$

$$\ddot{y} + 2\varepsilon\zeta_2\omega_2\dot{y} + \omega_2^2 y = \varepsilon\alpha_2 xy + R_2 \quad (2)$$

Where $R_1 = -\varepsilon G_1\dot{x}^3$, $R_2 = -\varepsilon G_2\dot{y}^3$, and x, y are the vibration amplitudes of the composite laminated rectangular thin plate for the first-order and the second-order modes, respectively, ω_1 and ω_2 the linear natural frequencies of the thin Plate, and Ω the excitations frequencies. f_1, f_2 are the amplitudes of the excitation forces, and ζ_1, ζ_2

are the linear damping coefficient of the plant, α_1, α_2, are the plant coupling constants, β is the non-linear coefficient, and G_1, G_2 are gain coefficients. We seek a second order uniform expansion for the solutions of equation (1) in the form:

$$x(t,\varepsilon) = x_0(T_0,T_1) + \varepsilon x_1(T_0,T_1) + \varepsilon^2 x_2(T_0,T_1) + O(\varepsilon^3) \quad (3)$$

$$y(t,\varepsilon) = y_0(T_0,T_1) + \varepsilon y_1(T_0,T_1) + \varepsilon^2 y_2(T_0,T_1) + O(\varepsilon^3) \quad (4)$$

where $T_n = \varepsilon^n t \quad ,(n = 0, 1)$, and the time derivatives became

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \dots \;, \qquad \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_1^2 + \dots, \quad (5)$$

where and ε is small a perturbation parameter and $0 < \varepsilon << 1$, T_0 is the fast time scale, T_1 is the slow time scales.
Substituting equations (3), (4) and (5) into equations (1) and (2) and equating the coefficients of same power of ε in both sides, we obtain the following set of ordinary differential equations:

Order ε^0:

$$\left(D_0^2 + \omega_1^2\right)x_0 = 0 \quad (6) \qquad \left(D_0^2 + \omega_2^2\right)y_0 = 0 \quad (7)$$

Order ε^1:

$$\left(D_0^2 + \omega_1^2\right)x_1 = -2D_0D_1x_0 - 2\zeta_1\omega_1D_0x_0 - \alpha_1x_0^3 + \beta y_0^2 + f_1\cos\Omega T_0 + f_2\cos 2\Omega T_0 - G_1(D_0x_0)^3 \quad (8)$$

$$\left(D_0^2 + \omega_2^2\right)y_1 = -2D_0D_1y_0 - 2\zeta_2\omega_2D_0y_0 + \alpha_2x_0y_0 - G_2(D_0y_0)^3 \quad (9)$$

Order ε^2:

$$\left(D_0^2 + \omega_1^2\right)x_2 = -D_1^2x_0 - 2D_0D_2x_0 - 2D_0D_1x_1 - 2\zeta_1\omega_1D_1x_0 - 2\zeta_1\omega_1D_0x_1 - 3\alpha_1x_1x_0^2 + 2\beta y_1y_0 - 3G_1\left(D_0x_0\right)^2 D_1x_0 - 3G_1\left(D_0x_0\right)^2 D_0x_1 \quad (10)$$

$$\left(D_0^2 + \omega_2^2\right)y_2 = -D_1^2y_0 - 2D_0D_2y_0 - 2D_0D_1y_1 - 2\zeta_2\omega_2D_1y_0 - 2\zeta_2\omega_2D_0y_1 + \alpha_2x_0y_1 + \alpha_2x_1y_0 - 3G_2\left(D_0y_0\right)^2 D_1y_0 - 3G_2\left(D_0y_0\right)^2 D_0y_1 \quad (11)$$

The general solution of equations (6) and (7) is given by

$$x_0(T_0,T_1) = A_0(\mathrm{T}_1)\exp(i\,\omega_1T_0) + \bar{A}_0(\mathrm{T}_1)\exp(-i\,\omega_1T_0) \quad (12)$$

$$y_0(T_0,\mathrm{T}_1) = B_0(\mathrm{T}_1)\exp(i\,\omega_2T_0) + \bar{B}_0(\mathrm{T}_1)\exp(-i\,\omega_2T_0) \quad (13)$$

where A_0, B_0 are unknown functions in T_1 at this level of approximation and can be determined by elimination the secular terms from the next order of perturbation. Substituting equations (12) and (13) into equations (8), (9) yields

$$\left(D_0^2+\omega_1^2\right)x_1=(-2i\,\omega_1 D_1 A_0-2\zeta_1 i\,\omega_1^2 A_0-3\alpha_1 A_0^2\,\bar{A}_0-3i\,\omega_1^3 A_0^2\,\bar{A}G_1)\exp(i\,\omega_1 T_0)$$
$$+\beta B_0^2\exp(2i\,\omega_2 T_0)+(G_1\,i\,\omega_1^3 A_0^3-\alpha_1 A_0^3)\exp(3i\,\omega_1 T_0)+\frac{f_1}{2}\exp(i\,\Omega T_0)$$
$$+\frac{f_2}{2}\exp(2i\,\Omega T_0)+\beta B_0\bar{B}_0+cc \qquad (14)$$

$$\left(D_0^2+\omega_2^2\right)y_1=(-2i\,\omega_2 D_1 B_0-2\zeta_2 i\,\omega_2^2 B_0-3i\,\omega_2^3 B_0^2\,\bar{B}_0 G_2)\exp(i\,\omega_2 T_0)+(G_2\,i\,\omega_2^3 B_0^3)\exp(3i\,\omega_2 T_0)$$
$$+\alpha_2 A_0 B_0\exp(i\,(\omega_1+\omega_2)T_0)+\alpha_2 A_0\bar{B}_0\exp(i\,(\omega_1-\omega_2)T_0)+cc \qquad (15)$$

The general solutions of equations (14) and (15) are:

$$x_1(T_0,T_1)=A_1(T_1)\exp(i\,\omega_1 T_0)+E_1\exp(2i\,\omega_2 T_0)+E_2\exp(3i\,\omega_1 T_0)+E_3\exp(i\,\Omega T_0)$$
$$+E_4\exp(2i\,\Omega T_0)+E_5+\text{cc} \qquad (16)$$

$$y_1(T_0,T_1)=B_1(T_1)\exp(i\,\omega_2 T_0)+E_6\exp(3i\,\omega_2 T_0)+E_7\exp(i\,(\omega_1+\omega_2)T_0)+E_8\exp(i\,(\omega_1-\omega_2)T_0)+\text{cc}$$
(17)

Substituting equations (12), (13), (16) and (17) into equations (10), (11) and solving the resulting equation we get:

$$x_2(T_0,T_1)=A_2(T_1)\exp(i\,\omega_1 T_0)+E_9\exp(3i\,\omega_1 T_0)+E_{10}\exp(2i\,\omega_1 T_0)+E_{11}\exp(2i\,\omega_2 T_0)$$
$$+E_{12}\exp(i\,\Omega T_0)+E_{13}\exp(2i\,\Omega T_0)+E_{14}\exp(5i\,\omega_1 T_0)+E_{15}\exp(2i\,(\omega_1+\omega_2)T_0)$$
$$+E_{16}\exp(i\,(2\omega_1+\Omega)T_0)+E_{17}\exp(2i\,(\omega_1+\Omega)T_0)+E_{18}\exp(2i\,(\omega_2-\omega_1)T_0)$$
$$+E_{19}\exp(i\,(\Omega-2\omega_1)T_0)+E_{20}\exp(2i\,(\Omega-\omega_1)T_0)+E_{21}\exp(4i\,\omega_2 T_0)$$
$$+E_{22}\exp(i\,(\omega_1+2\omega_2)T_0)+E_{23}\exp(i\,(\omega_1-2\omega_2)T_0)+E_{24}+cc \qquad (18)$$

$$y_2(T_0,T_1)=B_2(T_1)\exp(i\,\omega_2 T_0)+E_{25}\exp(3i\,\omega_2 T_0)+E_{26}\exp(i\,(\omega_1+\omega_2)T_0)+E_{27}\exp(i\,(\omega_1-\omega_2)T_0)$$
$$+E_{28}\exp(i\,(\omega_1+3\omega_2)T_0)+E_{29}\exp(i\,(2\omega_1+\omega_2)T_0)+E_{30}\exp(i\,(2\omega_1-\omega_2)T_0)$$
$$+E_{31}\exp(i\,(\omega_1-3\omega_2)T_0)+E_{32}\exp(i\,(\omega_2+3\omega_1)T_0)+E_{33}\exp(i\,(\omega_2+\Omega)T_0)$$
$$+E_{34}\exp(i\,(\omega_2+2\Omega)T_0)+E_{35}\exp(i\,(3\omega_1-\omega_2)T_0)+E_{36}\exp(i\,(\Omega-\omega_2)T_0)$$
$$+E_{37}\exp(i\,(2\Omega-\omega_2)T_0)+E_{38}\exp(5i\,\omega_2 T_0)+\text{cc} \qquad (19)$$

where $E_n,\ (\text{n}=1,...,38)$ are complex functions in T_1 and cc denotes the complex conjugate terms.

From the above derived solutions, the reported resonance cases are:

1) Primary resonance: $\Omega\cong\pm\omega_n,\ n=1,2.$

2) Sub-harmonic resonance: $\Omega\cong\pm2\omega_1$

3) Super-harmonic resonance: $\Omega\cong\pm\frac{1}{2}\omega_2$

4) Internal resonance: $\omega_1\cong\pm n\,\omega_2,\ n=2,3,\frac{1}{2},\frac{1}{3}.$

5) Simultaneous resonance: any combination of above resonance cases is considered as simultaneous resonance.

3. Stability analysis:

From the numerical solution at resonance cases obtained Table 1, we find that the worst is resonance case is the simultaneous resonance case $\Omega = \omega_1,\ \omega_1 = 2\omega_2$. So that we introduce the detuning parameters σ_1 and σ_2 according to the following:

$$\Omega = \omega_1 + \varepsilon\sigma_1, \omega_1 = 2\omega_2 + \varepsilon\sigma_2 \qquad (20)$$

Substituting equation (20) into equations (14) and (15) and eliminating the secular and small divisor terms from x_1 and y_1, we get the following:

$$2i\,\omega_1 D_1 A_0 = -2\zeta_1 i\,\omega_1^2 A_0 - 3\alpha_1 A_0^2\,\bar{A}_0 - 3i\,\omega_1^3 A_0^2\,\bar{A}G_1 + \beta B_0^2\exp(-i\,\sigma_2 T_1) + \frac{f_1}{2}\exp(i\,\sigma_1 T_1) \qquad (21)$$

$$2i\,\omega_2 D_1 B_0 = -2\zeta_2 i\,\omega_2^2\,B_0 - 3i\,\omega_2^3 B_0^2\,\bar{B}_0 G_2 + \alpha_2 A_0\,\bar{B}_0\exp(i\,\sigma_2 T_1) \qquad (22)$$

We express the complex function A_0, B_0 in the polar form as

$$A_0(T_1) = \frac{1}{2}a(T_1)\exp(i\,\theta_1(T_1)),\ B_0(T_1) = \frac{1}{2}b(T_1)\exp(i\,\theta_2(T_1)) \qquad (23)$$

where a, b, θ_1 and θ_2 are real.

Substituting equation (23) into equations (21) and (22) and separating real and imaginary part yields:

$$a' = -\zeta_1\omega_1 a - \frac{3}{8}\omega_1^2 a^3 G_1 + \frac{1}{4\omega_1}\beta b^2\sin\varphi_1 + \frac{f_1}{2\omega_1}\sin\varphi_2 \qquad (24)$$

$$a\theta_1' = \frac{3}{8\omega_1}\alpha_1 a^3 - \frac{1}{4\omega_1}\beta b^2\cos\varphi_1 - \frac{f_1}{2\omega_1}\cos\varphi_2 \qquad (25)$$

$$b' = -\zeta_2\omega_2 b - \frac{3}{8}\omega_2^2 b^3 G_2 + \frac{1}{4\omega_2}\alpha_2 ab\sin\varphi_1 \qquad (26)$$

$$b\theta_2' = -\frac{1}{4\omega_2}\alpha_2 ab\cos\varphi_1 \qquad (27)$$

where $\varphi_1 = 2\theta_2 - \theta_1 - \sigma_2 T_1,\ \varphi_2 = \sigma_1 T_1 - \theta_1$.

For the steady state solution $a' = b' = 0,\ \varphi_m' = 0; m = 1,2$. Then it follows from equations (24)-(27) that the steady state solutions are given by:

$$-\zeta_1\omega_1 a - \frac{3}{8}\omega_1^2 a^3 G_1 + \frac{1}{4\omega_1}\beta b^2\sin\varphi_1 + \frac{f_1}{2\omega_1}\sin\varphi_2 = 0 \qquad (28)$$

$$\frac{3}{8\omega_1}\alpha_1 a^3 - \frac{1}{4\omega_1}\beta b^2\cos\varphi_1 - \frac{f_1}{2\omega_1}\cos\varphi_2 - a\sigma_1 = 0 \qquad (29)$$

$$-\zeta_2\omega_2 b - \frac{3}{8}\omega_2^2 b^3 G_2 + \frac{1}{4\omega_2}\alpha_2 ab\sin\varphi_1 = 0 \qquad (30)$$

$$b\left(\frac{\sigma_1 + \sigma_2}{2}\right) + \frac{1}{4\omega_2}\alpha_2 ab\cos\varphi_1 = 0 \qquad (31)$$

From equations (28)-(31), we have the following cases:

Case 1: $a \neq 0$ and $b = 0$: in this case, the frequency response equation is given by:

$$\left(\frac{9}{64\omega_1^2}\alpha_1^2 + \frac{9}{64}\omega_1^4 G_1^2\right)a^6 + \left(\frac{3}{4}\zeta_1\omega_1^3 G_1 - \frac{3}{4\omega_1}\alpha_1\sigma_1\right)a^4 + \left(-\frac{3f_1}{8\omega_1^2}\alpha_1\cos\varphi_2 - \frac{3f_1}{8}\omega_1\,G_1\sin\varphi_2\right)a^3$$
$$+\left(\zeta_1^2\omega_1^2 + \sigma_1^2\right)a^2 + \left(\frac{f_1}{\omega_1}\sigma_1\cos\varphi_2 - f_1\zeta_1\sin\varphi_2\right)a + \frac{f_1^{\,2}}{4\omega_1^2} = 0 \qquad (32)$$

Case 2: $a = 0$ and $b \neq 0$: in this case, the frequency response equation is given by:

$$\frac{9}{64}\omega_2^4 G_2^2 b^6 + \frac{3}{4}\zeta_2\omega_2^3 G_2 b^4 + (\zeta_2^2\omega_2^2 + \frac{(\sigma_1+\sigma_2)^2}{4})b^2 = 0 \quad (33)$$

Case 3: $a \neq 0$ and $b \neq 0$: in this case, the frequency response equation are given by the following equations:

$$(\frac{9}{64\omega_1^2}\alpha_1^2 + \frac{9}{64}\omega_1^4 G_1^2)a^6 + (\frac{3}{4}\zeta_1\omega_1^3 G_1 - \frac{3}{4\omega_1}\alpha_1\sigma_1)a^4 + (-\frac{3f_1}{8\omega_1^2}\alpha_1\cos\varphi_2 - \frac{3f_1}{8}\omega_1 \mathrm{G}_1\sin\varphi_2$$
$$-\frac{3}{16}\omega_1 \mathrm{G}_1\beta \mathrm{b}^2\sin\varphi_1 - \frac{3}{16\omega_1^2}\alpha_1\beta \mathrm{b}^2\cos\varphi_1)a^3 + (\zeta_1^2\omega_1^2 + \sigma_1^2)a^2 + (\frac{f_1}{\omega_1}\sigma_1\cos\varphi_2 - f_1\zeta_1\sin\varphi_2$$
$$-\frac{1}{2}\zeta_1\beta \mathrm{b}^2\sin\varphi_1 + \frac{1}{2\omega_1}\sigma_1\beta \mathrm{b}^2\cos\varphi_1)a + (\frac{f_1^2}{4\omega_1^2} + \frac{1}{16\omega_1^2}\beta^2 \mathrm{b}^4 + \frac{f_1}{4\omega_1^2}\beta \mathrm{b}^2\cos(\varphi_1 - \varphi_2)) = 0$$

(34)

$$\frac{9}{64}\omega_2^4 G_2^2 b^6 + (\frac{3}{4}\zeta_2\omega_2^3 G_2 - \frac{3}{16}\omega_2\alpha_2 G_2 a\sin\varphi_1)b^4 + (\zeta_2^2\omega_2^2 + \frac{(\sigma_1+\sigma_2)^2}{4} - \frac{1}{2}\zeta_2\alpha_2 a\sin\varphi_1$$
$$+\frac{1}{4\omega_2}(\sigma_1+\sigma_2)\alpha_2 a\cos\varphi_1 + \frac{1}{16\omega_2^2}\alpha_2^2 a^2)b^2 = 0 \quad (35)$$

3.1. Linear solution:

Now to the stability of the linear solution of the obtained fixed let us consider A_0 and B_0 in the forms

$$A_0(T_1) = \frac{1}{2}(\mathrm{p}_1 - iq_1)\exp(i\,\delta_1 T_1) \text{ and } B_0(T_1) = \frac{1}{2}(\mathrm{p}_2 - iq_2)\exp(i\,\delta_2 T_1) \quad (36)$$

where p_1, p_2, q_1 and q_2 are real values and considering $\delta_1 = \sigma_1, \delta_2 = \sigma_2$.

Substituting equation (36) into the linear parts of equations (21), (22) and separating real and imaginary parts, the following system of equations are obtained:

Case 1: for the solution ($a \neq 0$ and $b = 0$), we get

$$p_1' + \zeta_1\omega_1 p_1 + \sigma_1 q_1 = 0 \quad (37)$$

$$q_1' - \sigma_1 p_1 + \zeta_1\omega_1 q_1 - \frac{f_1}{2\omega_1} = 0 \quad (38)$$

The stability of the linear solution is obtained from the zero characteristic equation

$$\begin{vmatrix} -(\lambda + \zeta_1\omega_1) & -\sigma_1 \\ \sigma_1 & -(\lambda + \zeta_1\omega_1) \end{vmatrix} = 0 \quad (39)$$

where $\lambda_{1,2} = -\zeta_1\omega_1 \pm i\,\sigma_1$

The linear solution is stable in this case if and only if $\zeta_1\omega_1 > \sigma_1$, ad otherwise it is unstable.

Case 2: for the solution ($a = 0$ and $b \neq 0$), we get

$$p_2' + \zeta_2\omega_2 p_2 + \sigma_2 q_2 = 0 \quad (40)$$

$$q_2' - \sigma_2 p_2 + \zeta_2\omega_2 q_2 = 0 \quad (41)$$

The stability of the linear solution is obtained from the zero characteristic equation

$$\begin{vmatrix} -(\lambda + \zeta_2\omega_2) & -\sigma_2 \\ \sigma_2 & -(\lambda + \zeta_2\omega_2) \end{vmatrix} = 0 \quad (42)$$

where $\lambda_{1,2} = -\zeta_2\omega_2 \pm i\,\sigma_2$

The linear solution is stable in this case if and only if $\zeta_2\omega_2 > \sigma_2$, ad otherwise it is unstable.

Case 3: for the solution ($a \neq 0$ and $b \neq 0$), we get

$$p_1' + \zeta_1\omega_1 p_1 + \sigma_1 q_1 = 0 \quad (43)$$

$$q_1' - \sigma_1 p_1 + \zeta_1\omega_1 q_1 - \frac{f_1}{2\omega_1} = 0 \quad (44)$$

$$p_2' + \zeta_2\omega_2 p_2 + \sigma_2 q_2 = 0 \quad (45)$$

$$q_2' - \sigma_2 p_2 + \zeta_2\omega_2 q_2 = 0 \quad (46)$$

The stability of the linear solution in this case is obtained from the zero characteristic equation

$$\begin{vmatrix} -(\lambda+\zeta_1\omega_1) & -\sigma_1 & 0 & 0 \\ \sigma_1 & -(\lambda+\zeta_1\omega_1) & 0 & 0 \\ 0 & 0 & -(\lambda+\zeta_2\omega_2) & -\sigma_2 \\ 0 & 0 & \sigma_2 & -(\lambda+\zeta_2\omega_2) \end{vmatrix} = 0 \quad (47)$$

after extract we obtain that

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0, \quad (48)$$

where

$r_1 = 2(\zeta_1\omega_1 + \zeta_2\omega_2)$, $r_2 = \sigma_2^2 - \sigma_1^2 + \zeta_1^2\omega_1^2 + 4\zeta_1\zeta_2\omega_1\omega_2 + \zeta_2^2\omega_2^2$,

$r_3 = 2\zeta_1\omega_1(\zeta_2^2\omega_2^2 + \sigma_2^2) + 2\zeta_2\omega_2(\zeta_1^2\omega_1^2 - \sigma_1^2)$, $r_4 = (\zeta_1^2\omega_1^2 - \sigma_1^2)(\zeta_2^2\omega_2^2 + \sigma_2^2)$

According to the Routh-Huriwitz criterion, the above linear solution is stable if the following are satisfied: $r_1 > 0,\ r_1r_2 - r_3 > 0,\ r_3(r_1r_2 - r_3) - r_1^2 r_4 > 0,\ r_4 > 0.$

3.2. Non-linear solution:

To determine the stability of the fixed points, one lets

$$a = a_{10} + a_{11},\ b = b_{10} + b_{11} \text{ and } \varphi_m = \varphi_{m0} + \varphi_{m1}, (m = 1, 2), \quad (49)$$

where a_{10}, b_{10} and φ_{m0} are the solutions of equations (28-31) and $a_{11}, b_{11}, \varphi_{m1}$ are perturbations which are assumed to be small compared to a_{10}, b_{10} and φ_{m0}. Substituting equation (49) into equations (24-27), using equations (28-31) and keeping only the linear terms in $a_{11}, b_{11}, \varphi_{m1}$ we obtain:

Case 1: for the solution ($a \neq 0$ and $b = 0$), we get:

$$a_{11}' = -(\zeta_1\omega_1 + \frac{9}{8}\omega_1^2 G_1 a_{10}^2)a_{11} + (\frac{f_1}{2\omega_1}\cos\varphi_{20})\varphi_{21} \quad (50)$$

$$\varphi_{21}' = (\frac{\sigma_1}{a_{10}} - \frac{9\alpha_1 a_{10}}{8\omega_1})a_{11} - (\frac{f_1}{2\omega_1 a_{10}}\sin\varphi_{20})\varphi_{21} \quad (51)$$

The stability of a given fixed point to a disturbance proportional to $\exp(\lambda t)$ is determined by the roots of

$$\begin{bmatrix} -\zeta_1\omega_1 - \frac{9}{8}\omega_1^2 G_1 a_{10}^2 & \frac{f_1}{2\omega_1}\cos\varphi_{20} \\ \frac{\sigma_1}{a_{10}} - \frac{9\alpha_1 a_{10}}{8\omega_1} & -\frac{f_1}{2\omega_1 a_{10}}\sin\varphi_{20} \end{bmatrix} = 0 \quad (52)$$

Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix (52) are less than zero.

Case 2: for the solution ($a \neq 0$ and $b \neq 0$), we get:

$$a_{11}' = -(\zeta_1\omega_1 + \frac{9}{8}\omega_1^2 G_1 a_{10}^2)a_{11} + (\frac{\beta b_{10}}{2\omega_1}\sin\varphi_{10})b_{11} + (\frac{\beta b_{10}^2}{4\omega_1}\cos\varphi_{10})\varphi_{11} + (\frac{f_1}{2\omega_1}\cos\varphi_{20})\varphi_{21} \quad (53)$$

$$\varphi_{21}' = (\frac{\sigma_1}{a_{10}} - \frac{9\alpha_1 a_{10}}{8\omega_1})a_{11} + (\frac{\beta b_{10}}{2\omega_1 a_{10}}\cos\varphi_{10})b_{11} - (\frac{\beta b_{10}^2}{4\omega_1 a_{10}}\sin\varphi_{10})\varphi_{11} - (\frac{f_1}{2\omega_1 a_{10}}\sin\varphi_{20})\varphi_{21} \quad (54)$$

$$b_{11}' = (\frac{\alpha_2}{4\omega_2}b_{10}\sin\varphi_{10})a_{11} - (\zeta_2\omega_2 + \frac{9}{8}\omega_2^2 b_{10}^2 G_2 - \frac{\alpha_2}{4\omega_2}a_{10}\sin\varphi_{10})b_{11} + (\frac{\alpha_2}{4\omega_2}a_{10}b_{10}\cos\varphi_{10})\varphi_{11} \quad (55)$$

$$\varphi_{11}' = (\frac{\sigma_1}{a_{10}} - \frac{9\alpha_1 a_{10}}{8\omega_1} - \frac{\alpha_2}{2\omega_2}\cos\varphi_{10})a_{11} - (\frac{\alpha_2}{2\omega_2 b_{10}}a_{10}\cos\varphi_{10} + \frac{\sigma_1+\sigma_2}{b_{10}} - \frac{\beta b_{10}}{2\omega_1 a_{10}}\cos\varphi_{10})b_{11}$$

$$+ (\frac{\alpha_2}{2\omega_2}a_{10}\sin\varphi_{10} - \frac{\beta b_{10}^2}{4\omega_1 a_{10}}\sin\varphi_{10})\varphi_{11} - (\frac{f_1}{2\omega_1 a_{10}}\sin\varphi_{20})\varphi_{21} \quad (56)$$

The stability of a particular fixed point with respect to perturbations proportional to $\exp(\lambda t)$ depends on the real parts of the roots of the matrix. Thus, a fixed point given by equations (56)-(59) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

4. Numerical results:

The behavior of the given system of equations (1), (2) has been solved numerically applying Runge-Kutta 4th order method [24, 25]. Fig. 1 illustrates the response and phase-plane for the non- resonant system at some practical values of the equations parameters. From this fig. we can see that the system is stable with the steady state amplitude x and y are 0.01 and 0.03 respectively, and the phase plane shows the system is stable with multi limit cycles

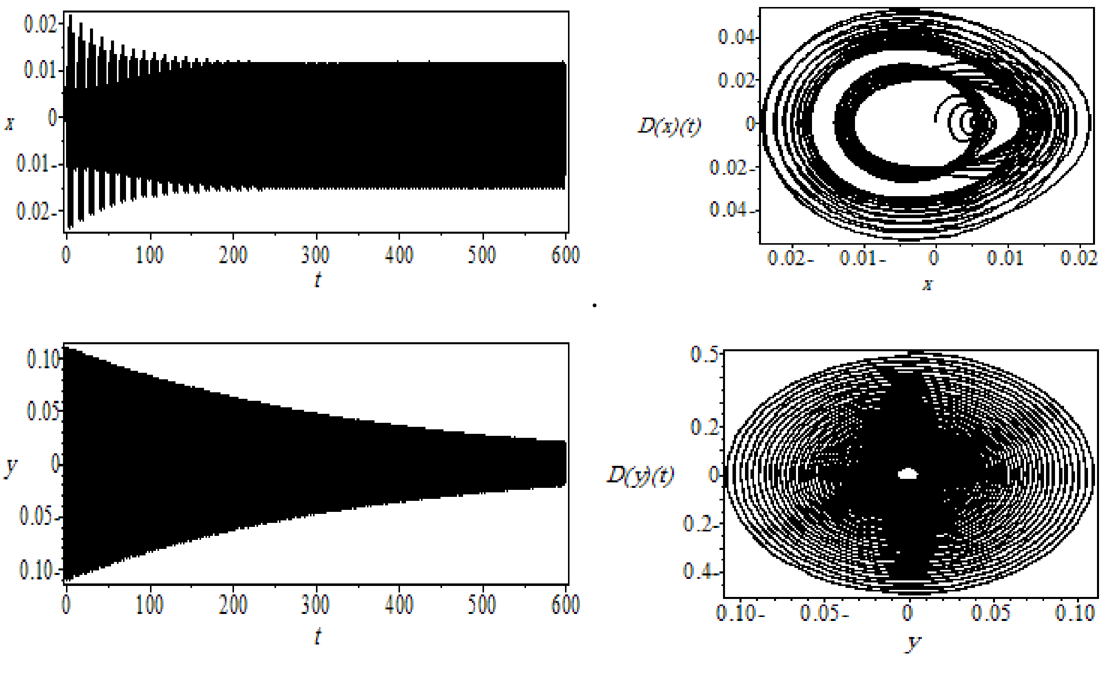


Fig. 1. The basic case of the system without controller.

4.1 Resonance Cases:

Some of the deduced resonance cases of the plant without the absorber are studied numerically as shown in Table 1. From this table, we see that the amplitude increasing at the resonance cases and the worst case is the simultaneous resonance case when $\Omega = \omega_1,\ \omega_1 = 2\omega_2$, which the amplitudes are increased to about 1000% compared with the basic case shown in fig. 1. If the control is added the amplitudes x and y are reduced 0.04 and 0.01 respectively compared with the system without absorber shown in Fig.2, which means that the system needs to reduced the amplitude of vibration or controlled, in Fig. 3.

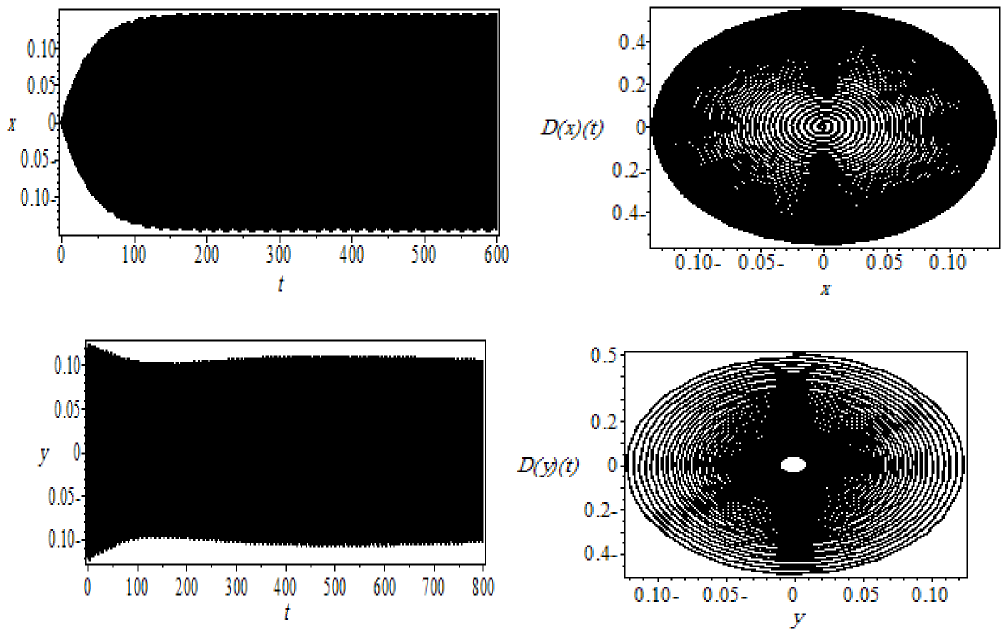


Fig. 2. System behavior without controller at simultaneous resonance

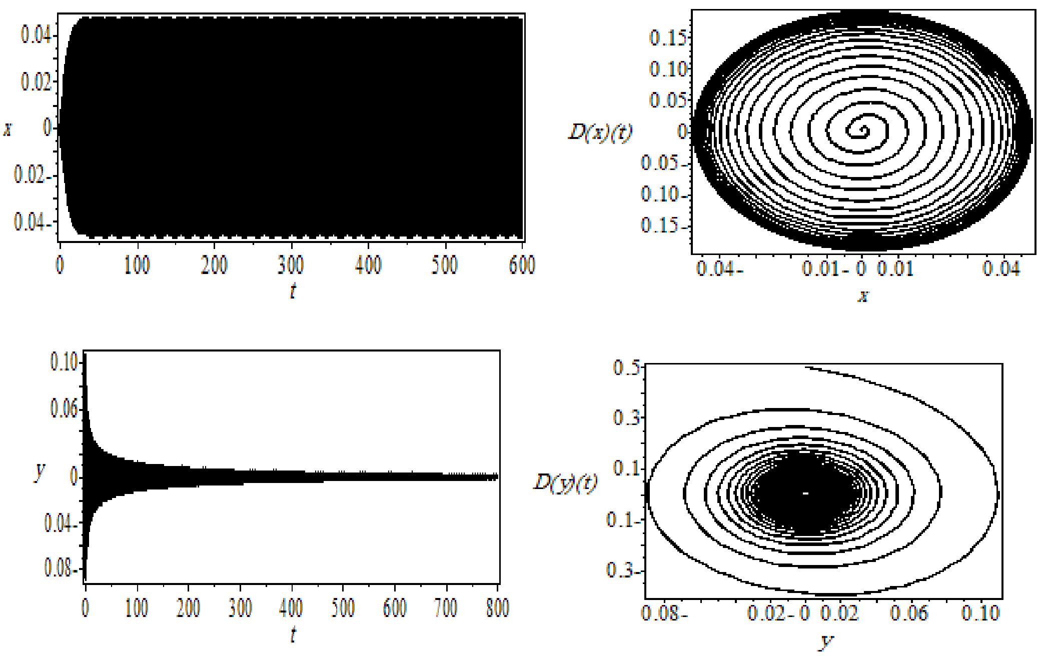


Fig. 3. System behavior with controller at simultaneous resonance

4.2 Effect of the Controller:

Fig. 3, illustrates the results when the controller is effective, when $\Omega = \omega_1$, and $\omega_1 = 2\omega_2$.The effectiveness of the controller is Ea (Ea = steady state amplitude of the main system without controller/ steady state amplitude of the main system with controller) are about 2.5 and 12.

Table 1. Resonance cases

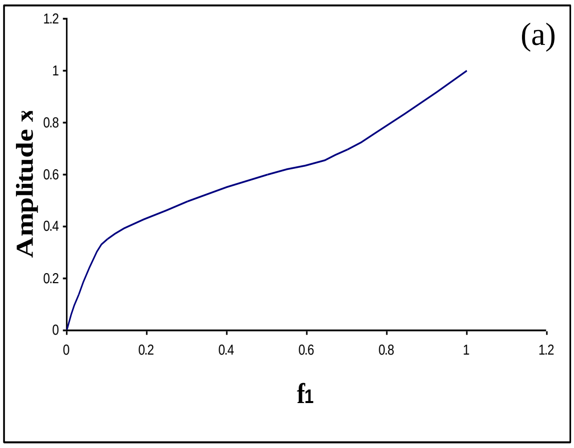
Resonance cases	Amplitude of x	Amplitude of y

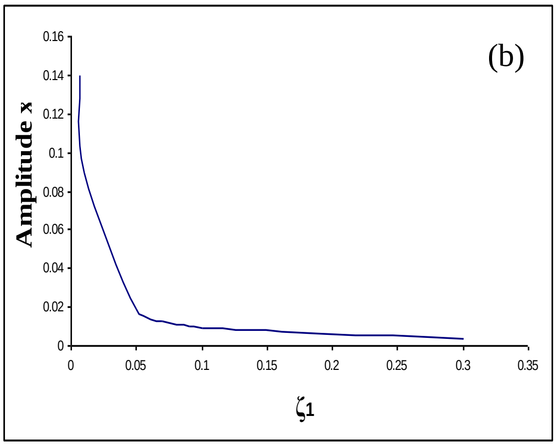
	Without control	With control	E_a	Without control	With control	E_a
$\omega_1=3\omega_2$	0.01	0.01	1	0.3	0.05	6
$\Omega=\omega_1$	0.3	0.12	2.5	0.04	0.01	4
$_2\omega=\Omega$	0.004	0.004	1	0.03	0.01	3
$_1\omega 2=\Omega$	0.003	0.0028	1.1	0.05	0.01	5
$_2\omega = \Omega 2$	0.02	0.018	1.1	0.03	0.01	3
$_2\omega={}_1\omega 3$	0.006	0.005	1.2	0.05	0.01	5
$_2\omega 2={}_1\omega$	0.005	0.004	1.25	0.05	0.01	5
$\Omega= \omega_1,\ \omega_1 =2\omega_2$	0.025	.001	2.5	0.6	0.05	12

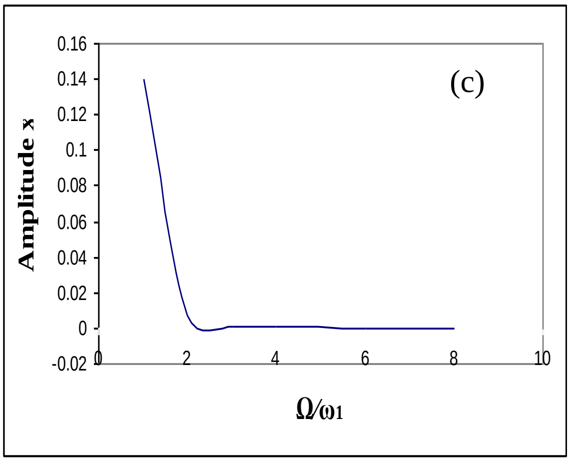
4.3 Effect of Parameters:

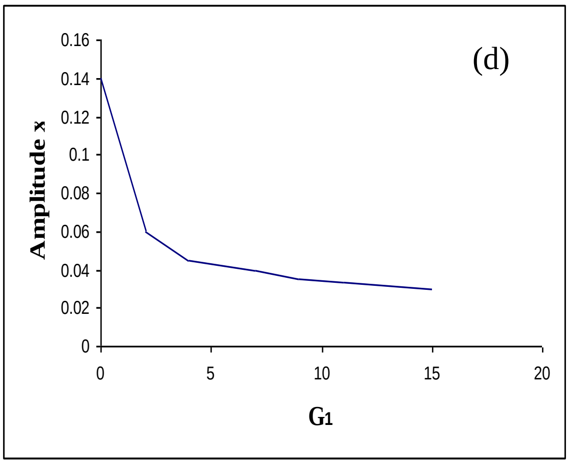
The amplitude of the x is monotonic increasing function of the excitation force f_1 as shown in Fig. 4a. But the amplitude of the system is monotonic decreasing function of the linear damping coefficient ζ_1 as shown in Fig. 4b. and the amplitude of the system is monotonic decreasing function of the ratio Ω/ω_1 as shown in Fig. 4c. and the amplitude of the system is monotonic decreasing function of the gain coefficient G_1 as shown in Figs. 4d.

The amplitude of the y is monotonic increasing function of the plant coupling constants α_2, as shown in fig. 4e. But amplitude of the system is monotonic decreasing function of the gain coefficient G_2 as shown in Fig. 4f.









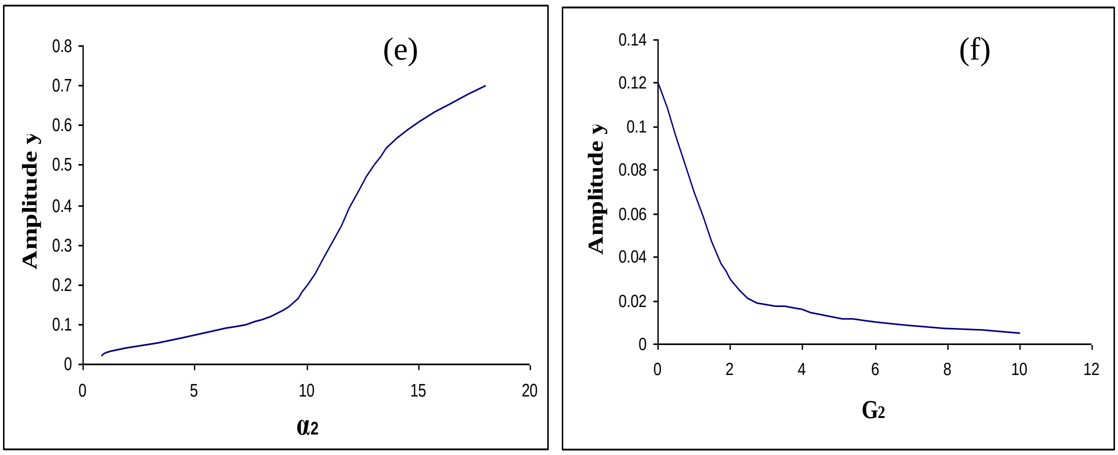


Fig. 4. Effect of parameters

4.5 Response curves:

The frequency response equations (32), (33), (34) and (35) are nonlinear algebraic equations of a, b. These equations are solved numerically as shown in Figs. 5-8. From case 1 where $a \neq 0,\ b = 0$: Fig .5, shows that the steady state amplitudes of the system are monotonic decreasing function in α_1. But monotonic increasing function in ω_1. Fig. 6, shows the force response equation (32) is a nonlinear algebraic equation of a, which are solved numerically of the amplitude against the excitation force amplitude f_1. From this Fig. the steady state amplitude is monotonic increasing functions in ω_1, σ_1 as shown in Figs. 6a, 6b. The steady state amplitude is monotonic decreasing functions in G_1, ζ_1 as shown in Fig. 6c, 6d. which are a good agreement with the frequency response curves. From case3, where $a \neq 0,\ b \neq 0$: Fig. 7, shows that the steady state amplitudes of the system are monotonic increasing functions in ω_1, But monotonic decreasing functions in α_1. From Fig. 8, shows that the steady state amplitudes of the system are monotonic decreasing functions in ζ_2, G_2, α_2.

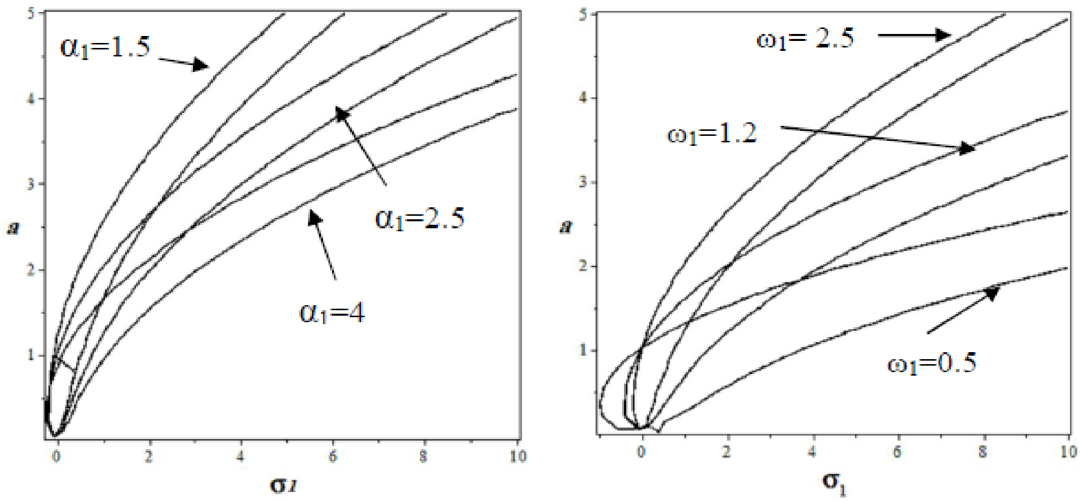


Figure 5: Frequency Response curves ($a \neq 0$ and b=0)

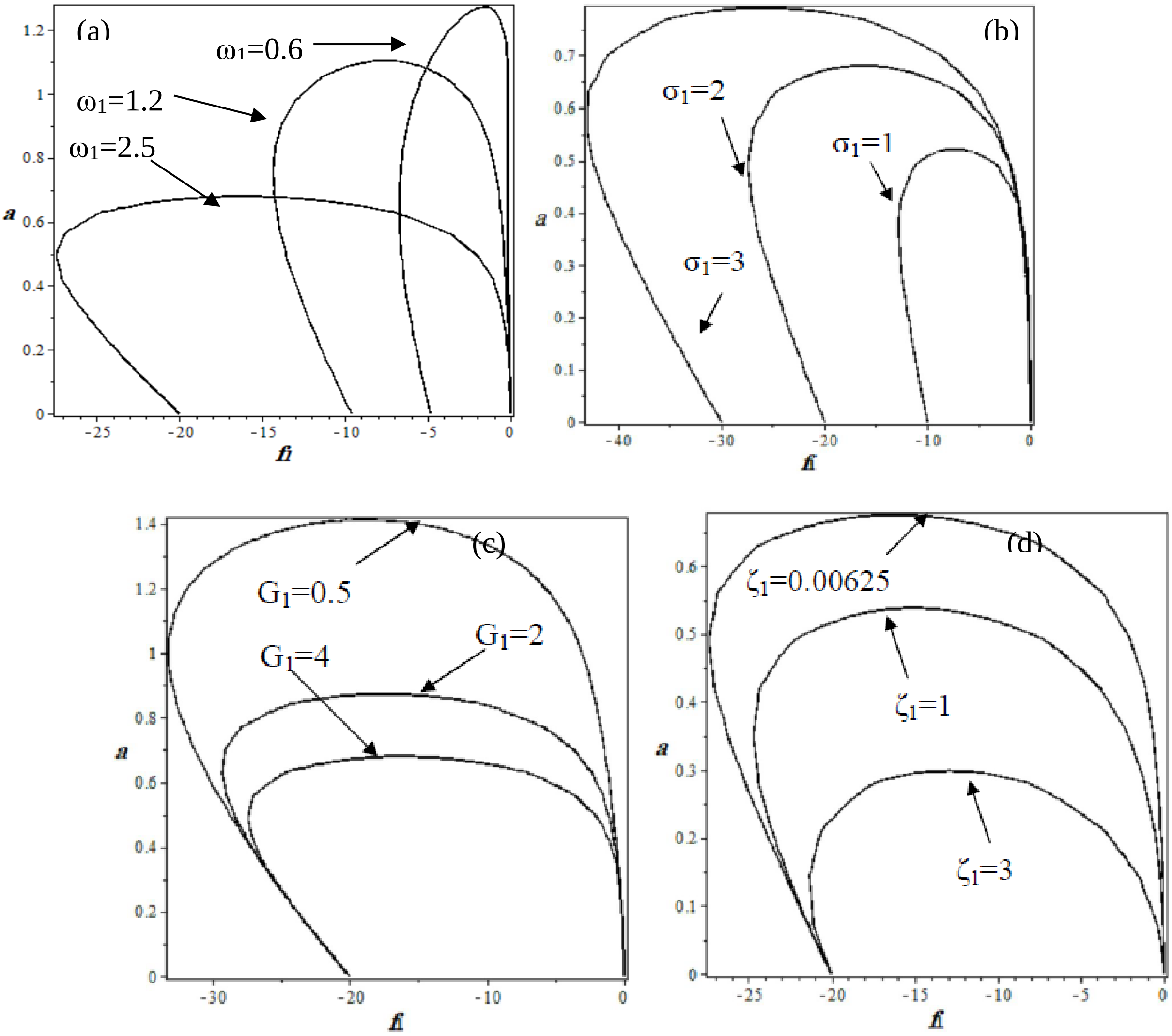


Fig. 6. Excitation response curve ($a \neq 0$ *and* $b = 0$)

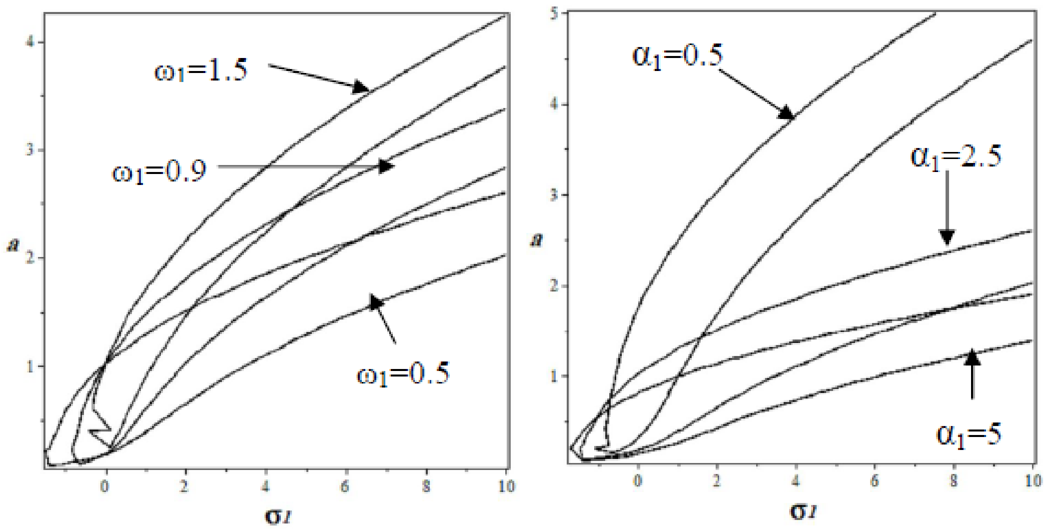


Fig. 7. Response curves ($a \neq 0$ *and* $b \neq 0$)

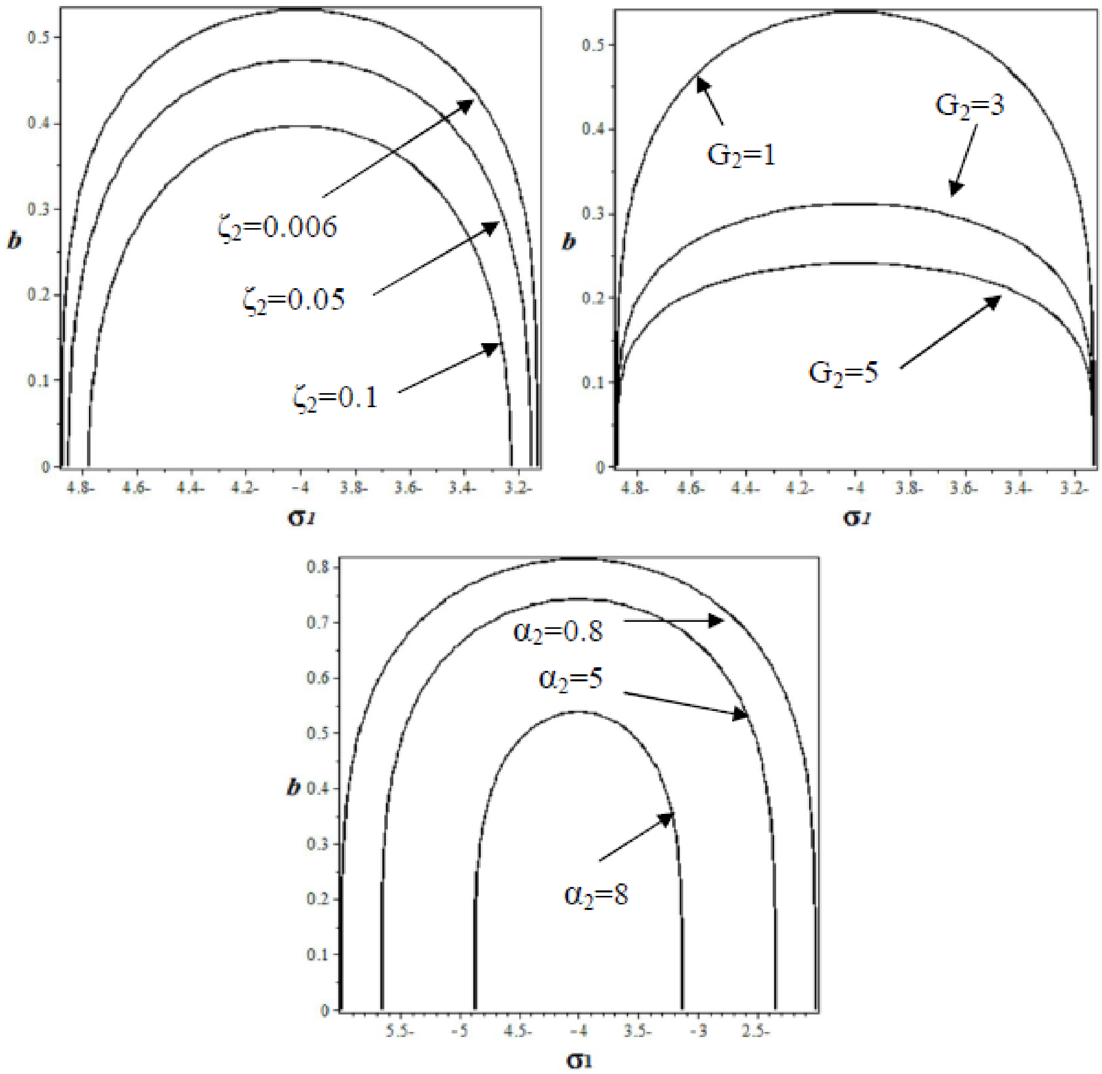


Fig. 8. Response curves (*a≠0 and b≠*0)

5. Conclusions:

The vibrations of a coupled second order nonlinear differential equations having

1- The worst resonance case is the simultaneous resonance case when $\Omega = \omega_1, \ \ \omega_1 = 2\omega_2$, which the amplitudes are increased to about 1000% compared with the basic case .

2- The control can reduced the amplitudes x and y to about 0.04 and 0.01 respectively compared with the system without absorber.

3- The amplitude of the x is monotonic increasing function of the excitation force f_1. But it is decreasing function of the gain coefficient G_1, ζ_1, and Ω / ω_1 .

4- The amplitude of the y is monotonic decreasing function of the coefficient gain G_2. But it is monotonic increasing function of the plant coupling constants α_2.

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