التقريب التحليلي لنظام مستمر غير خطي ضعيف باستخدام تقنية اضطرابات متعددة النطاق

د . سميرة مختار الطاهر سليمان (*)، د . حنان محمد حسين الشيباني
د . فاطمة مولود علي الصويعي .
كلية العلوم ـ جامعة الزاوية

مستخلص:

في هذه الورقة، يتم التحقيق في اهتزازات لوحة مستطيلة مغلقة ودراستها. يتم وصف النظام من خلال المعادلة التفاضلية غير الخطية من درجة الأولى يتعرض لقوة خارجية متعددة الإثارة. يتم الحصول على التحكم في الاهتزاز باستخدام طريقة تأخير الوقت. يتم تطبيق تقنية اضطرابات متعددة النطاق لدراسة الحل التقريبي للنظام المعطى. ويتم دراسة تأثير البامترات المختلفة على النظام عدديا. ينتج من ذلك استقرار النظام بالقرب من حالة الرنين عند تطبيق معادلة الاستجابة الترددية. وفي الحل الخطي إذا كانت موجبة فإن الحلول مستقرة.

Analytical approximation of weakly nonlinear continuous systems using multiple scale perturbation technique

Samira M. Soleman, Hanan M. Shiabani & Fatma M. Sewaee
Department of Mathematics, Faculty of Science, EL-Zawia University, Libya.

Abstract:

In this paper, vibrations of the composite laminated rectangular plate are investigated and studied. The system is described by nonlinear differential equation a one-degree-of-

(*) Email: sa.sulayman.ly@zu.edu.ly

freedom system that subjected to multi parametric excitation force. Vibration control is obtained using of the time delay method. The multiple scale perturbation technique (MSPT) is applied to study the approximate solution of the given system. The effects of different parameter's on the main system behavior are studied numerically. This results in the system being stable near the resonance case applying the frequency-response equations, and in the linear solution if μ is positive then the solutions are stable.

Keywords: Vibrations control; Feedback control; Time delay; Stability

1- Introduction:

In the previous decades, vibration and noise of the systems have been studied extensively. Recently, Limitations associated with the standards related to the gear system and worldwide competition increase the importance of this concern. The backlash effect associated with the variable stiffness has been analyzed via the harmonic balance method by Kahraman and Singh [1]. Elnaggar and Khalil [2], studied The response of nonlinear controlled system under an external excitation via time delay state feedback. El-Gohary et al. [3] studied vibration suppression of a dynamical system to multi-parametric excitations via time-delay absorber. In order to overcome such a drawback of the autoparametric vibration absorber, they proposed a new type of nonlinear dynamics vibration absorber in[4], and theoretically and experimentally confirmed the validity for the primary resonance.

A complete literature overview on the mathematical modeling of the gear systems have been provided by Ozguven and Houser [5]. Yabuno et al. Showed that an autoparametric vibration absorber cannot stabilize the parametric resonance, but the autoparametric coupling produces chaotic motions in the main system. Furthermore, they proposed an active actuation pendulum as the vibration absorber which is based on open- loop resonance cancellation [6]. Many structural materials obey this relationship, but the origin of this type of nonlinearity can also be geometrical or associated with a physical configuration, as in the case of a pendulum performing small vibrations, snap-through mechanisms, beam cables, electrical circuits, by Kovacic [7].

Thomsen [8]. However, the related analysis was developed based on a classical single degree of freedom model. Saeed, El-Ganini, and Eissa [9], studied nonlinear time delay saturation-based controller for suppression of nonlinear beam vibrations.

Dynamic transmission error (DTE) caused by tooth deflection is another displacement-type source of vibrations in spur gear pairs by Gregory et al. [10]. O'Malley and Kirkinis [11] showed that a multi-scale method may often be preferable for solving singularly perturbed problems than the method of matched asymptotic expansions.

The technique of delayed feedback control vibration absorber is a new technique of vibration suppression. In fact, time delay in control system is derived from measurements of system states, by physical properties of the equipment used for control and transport delay, by performing on-line computation, filtering and processing data and by calculating and executing the control forces as required. Therefore, the studies of control system with time delay have been developed in various research fields, such as mechanics [12].

In the early works a basic theory was developed by Mark [13] to find an analytical expression for the static transmission error, which was considered the main parameter to control the gear dynamics. Ganaini and Elgohary [14], studied Vibration suppression via time-delay absorber described by non-linear differential equations. Gao and Chen [15], studied Active vibration control for a Bilinear system with nonlinear velocity time-delayed feedback.

Recently, Shen et al. [16] used incremental harmonic balance method to analyze nonlinear dynamics of a spur gear system including time varying stiffness and backlash. Cheung et al. [17] presented a new modified Lindstedt-Poincare method by following the concept of the introduction of a new small parameter, enabling a strongly nonlinear system corresponding to the original parameter ε to be transformed into a weakly nonlinear system. Their approach was only applied to the doffing oscillator.

Amer and Abd Elsalam [18], studied vibration control of the main system using absorber at the simultaneous primary and internal resonance . Amer and Ahmed [19], studied Vibration control of a nonlinear dynamical system with time varying stiffness subjected to multi external forces. Yingli, Daolin, Yiming, and Jing [20], studied Dynamic effects of delayed feedback control on nonlinear vibration floating raft systems.

2. Mathematical Analysis with Time Delay:

In this section, we study the following equation:

$$\ddot{q} + 2\varepsilon\mu\dot{q} + \omega^2 q + \varepsilon\alpha_1 q^2 + \varepsilon\alpha_2 q^3 = \varepsilon q f \cos\Omega t \quad \textbf{(1)}$$

where, q is the vibration amplitudes of the composite laminated rectangular thin plate for the first-order and the mode, μ the modal damping coefficient, ω the linear natural frequency of the thin Plate, and Ω the excitation frequency. f is the amplitude of excitation force, $\alpha_i \ (i=1,2)$ are non-linear coefficients. We seek a second order uniform expansion for the solutions of equation (1) in the form:

$$q(t,\varepsilon) = q_0(T_0,T_1) + \varepsilon q_1(T_0,T_1) + \varepsilon^2 q_2(T_0,T_1) + O(\varepsilon^3)$$

(2)

where $T_n = \varepsilon^n t, \ (n = 0,1)$**, and the time derivatives became**

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \dots \ , \qquad \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_1^2 + \dots,$$

Substituting equations (2), (3) into equations (1) and equating the coefficients of same power of ε in both sides, we obtain the following set of ordinary differential equations:

Order ε^0:

$$\left(D_0^2+\omega^2\right)q_0=0 \tag{4}$$

Order ε^1:

$$\left(D_0^2+\omega^2\right)q_1=-2D_0D_1q_0-2\mu D_0q_0-\alpha_1q_0^2-\alpha_2q_0^3+fq_0\cos\Omega T_0 \tag{5}$$

Order ε^2:

$$\left(D_0^2+\omega^2\right)q_2=-2D_0D_1q_1-D_1^2q_0-2\mu D_0q_1-2\mu D_1q_0-2\alpha_1q_0q_1-3\alpha_2q_0^2q_1+fq_1\cos\Omega T_0 \tag{6}$$

The general solution of equations (4) is given by

$$q_0(T_0,T_1)=A(\mathrm{T}_1)\exp(i\,\omega T_0)+\bar{A}(\mathrm{T}_1)\exp(-i\,\omega T_0) \tag{7}$$

Where A is unknown function in T_1 at this level of approximation and can be determined by elimination the secular terms from the next order of perturbation.

Substituting equation (7) into equation (5) yields

$$\left(D_0^2+\omega^2\right)q_1=(-2i\,\omega D_1A-2\mu i\,\omega A-3\alpha_2A^2\bar{\mathrm{A}})\exp(i\,\omega T_0)-\alpha_1A^2\exp(2i\,\omega T_0)-\alpha_2A^3\exp(3i\,\omega T_0)$$
$$+\frac{f}{2}A\exp(i\,(\omega+\Omega)T_0)-\alpha_1\,\mathrm{A}\bar{\mathrm{A}}+\mathrm{cc} \tag{8}$$

The general solution of equation (8) is:

$$q_1=\frac{\alpha_1A^2}{3\omega^2}\exp(2i\,\omega T_0)+(\frac{\alpha_2A^3}{8\omega^2})\exp(3i\,\omega T_0)-\frac{fA}{2\Omega(2\omega+\Omega)}\exp(i\,(\omega+\Omega)T_0)-\alpha_1\,\mathrm{A}\bar{\mathrm{A}}+cc \tag{9}$$

Substituting equations (7) and (9) into equations (6) and solving the resulting equation we get:

$$\mathrm{q}_2=\mathrm{E}_1\exp(2i\,\omega T_0)+\mathrm{E}_2\exp(3i\,\omega T_0)+\mathrm{E}_3\exp(4i\,\omega T_0)+\mathrm{E}_4\exp(5i\,\omega T_0)+\mathrm{E}_5\exp(i\,(\omega+\Omega)T_0)$$
$$+\mathrm{E}_6\exp(i\,(2\omega+\Omega)T_0)+\mathrm{E}_7\exp(i\,(3\omega+\Omega)T_0)+\mathrm{E}_8\exp(i\,(\Omega-\omega)T_0)+\mathrm{E}_9\exp(i\,(\omega+2\Omega)T_0)$$
$$+\mathrm{E}_{10}\exp(i\,\Omega T_0)+\mathrm{E}_{11}+cc \tag{10}$$

where E_n, $(\mathrm{n}=1,\ldots,11)$ are complex functions in T_1 and cc denotes the complex conjugate terms.

From the above derived solutions, the reported resonance cases are:

1) Primary resonance: $\Omega\cong\omega$.

2) Sub-harmonic resonance: $\Omega \cong j\,\omega,\ j = 2, 3.$

3) Super- harmonic resonance: $\Omega \cong \frac{1}{2}\omega.$

Here it must be explain what the research will exactly studies, that is we will study the stability system, the linear and nonlinear system and in the end we discuss some numerical results about our system.

2.1 Stability analysis:

We introduce the detuning parameter σ according to the following:

$$\Omega = 2\omega + \varepsilon\sigma \quad \textbf{(11)}$$

Substituting equation (11) into equation (8) and eliminating the secular and small divisor terms from q, we get the following:

$$D_1 A = -\mu A + \frac{3\alpha_2 i}{2\omega} A^2 \bar{A} - \frac{f\ \bar{A} i}{4\omega} \exp(i\,\sigma T_1) \quad \textbf{(12)}$$

We express the complex function A in the polar form as

$$A(T_1) = \frac{1}{2} a(T_1) \exp(i\,\gamma(T_1)) \quad \textbf{(13)}$$

where a and γ are real.

Substituting equation (13) into equations (12) and separating real and imaginary part yields:

$$a\theta' = \sigma a - \frac{3\alpha_2 a^3}{4\omega} + \frac{fa}{2\omega}\cos\theta \quad \textbf{(14)}$$

$$a' = -\mu a + \frac{fa}{4\omega}\sin\theta \quad \textbf{(15)}$$

where $\theta = \sigma T_1 - 2\gamma$.

For the steady state solution $a'=0,\ \theta'=0$. Then it follows from equations (14) and (15) that the steady state solutions are given by

$$\sigma a-\frac{3\alpha_2 a^3}{4\omega}+\frac{fa}{2\omega}\cos\theta=0 \tag{16}$$

$$-\mu a+\frac{fa}{4\omega}\sin\theta=0 \tag{17}$$

Then we have

$$\frac{9\alpha_2^2}{16\omega^2}a^6+-\frac{3\alpha_2\sigma}{2\omega}a^4+(4\mu^2+\sigma^2-\frac{f^{\,2}}{4\omega^2})a^2=0 \tag{18}$$

2.2 Linear solution:

Now to the stability of the linear solution of the obtained fixed let us consider A in the forms

$$A(T_1)=\frac{1}{2}(p_1-ip_2)\exp(i\,\delta T_1) \tag{19}$$

where p_1, p_2 are real values, and considering $\delta=\frac{1}{2}\sigma.$

Substituting equation (19) into the linear parts of equations (12) and separating real and imaginary parts, the following system of equations are obtained:

$$p_1'=-\mu p_1+(\frac{-\sigma}{2}+\frac{f}{4\omega})p_2 \tag{20}$$

$$p_2'=(\frac{\sigma}{2}+\frac{f}{4\omega})p_1-\mu p_2 \tag{21}$$

The stability of the linear solution is obtained from the zero characteristic equation

$$\begin{vmatrix} -(\lambda+\mu) & (\frac{-\sigma}{2}+\frac{f}{4\omega}) \\ (\frac{\sigma}{2}+\frac{f}{4\omega}) & -(\lambda+\mu) \end{vmatrix} = 0 \qquad \textbf{(22)}$$

where $\lambda_{1,\,2} = -\mu \pm \frac{1}{4\omega}\sqrt{f^{\,2} - 4\omega^2\sigma^2}$

since μ is positive then the solutions are stable.

2.3 Non-linear solution:

To determine the stability of the fixed points, one lets

$$a = a_0 + a_1 \ \textbf{and} \ \theta = \theta_0 + \theta_1 \qquad \textbf{(23)}$$

where a_0 and θ_0 are the solutions of equations (16) ,(17) and a_1, θ_1 are perturbations which are assumed to be small compared to a_0 and θ_0. Substituting equation (23) into equations (14), (15), using equations (16), (17) and keeping only the linear terms in a_1, θ_1 we obtain:

$$a_1' = (-\mu + \frac{f}{4\omega}\sin\theta_0)\,\mathrm{a}_1 + (\frac{fa_0}{4\omega}\cos\theta_0)\theta_1 \qquad \textbf{(24)}$$

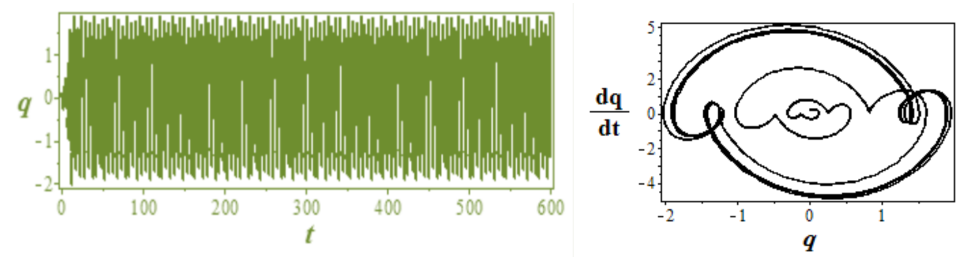
$$\theta_1' = (\frac{\sigma}{a_0} - \frac{9\alpha_2 a_0}{4\omega} + \frac{f}{2\omega a_0}\cos\theta_0)\,\mathrm{a}_1 + (\frac{f}{2\omega}\sin\theta_0)\theta_1 \qquad \textbf{(25)}$$

$$\begin{bmatrix} -\mu + \frac{f}{4\omega}\sin\theta_0 & \frac{fa_0}{4\omega}\cos\theta_0 \\ \frac{\sigma}{a_0} - \frac{9\alpha_2 a_0}{4\omega} + \frac{f}{2\omega a_0}\cos\theta_0 & \frac{f}{2\omega}\sin\theta_0 \end{bmatrix} = 0 \qquad \textbf{(26)}$$

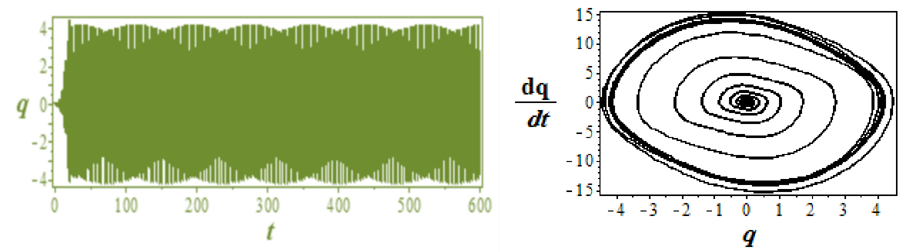
Consequently, a non-trivial solution is stable if and only if the real parts of both eigenvalues of the coefficient matrix (26) are less than zero.

2.4 Numerical results:

The Runge-Kutta of fourth order method by using Maple 16 software was applied to determine the numerical solution of the given system of (1) Fig.1 shows that the amplitude is decreasing function of the time and the system is stable, and the phase plane is limit cycles which is consider that is basic case. Fig.2 shows the system response at sub-harmonic resonance case $\Omega \cong 2\omega$, from this figure we see that the amplitude of the system is increased to about 2.7 times of the amplitude of the basic case shows in Fig. 1.



0.3, f=6.6, α_1 =0.02, α_2 =0.2, ε=0.01, Ω=2.5) μ=1.8,(ω=Fig.1. Basic case



. $\Omega \cong 2\omega$ **Fig.2. Sub-harmonic resonance case**

i. Effect of Parameters:

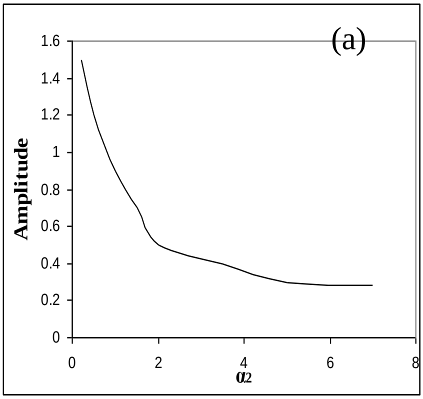
The amplitude of the system is monotonic decreasing functions of the non-linear coefficient α_2 , and the amplitude of the system is monotonic decreasing function of the damping coefficient μ , as shown in Fig. 5a, 5b. But the amplitude of the system is monotonic increasing function of the excitation force f as shown in Fig. 5c.

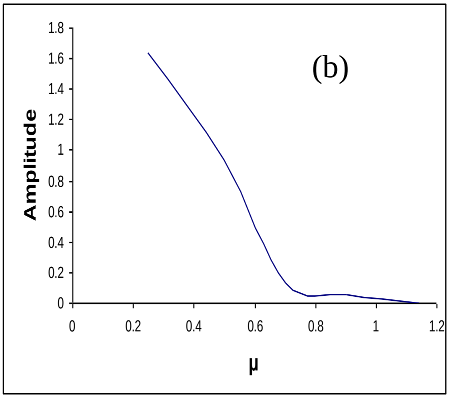
ii. Frequency Response curves:

The frequency response equations (18) is nonlinear algebraic equations of a. This equation is solved numerically as shown in Figs. 6, shows that the steady state amplitude of the system is monotonic decreasing function in ω. But monotonic increasing functions in f · And increasing values of the α_2 leads to steady state amplitude is bent to the right. But the steady state amplitude of the system is monotonic decreasing function in μ.

iii. Excitation response curves:

Figs. 7, shows the force response equation (18) is a nonlinear algebraic equation of a, which is solved numerically of the amplitude against the excitation force amplitude f. From this Figs. the steady state amplitude is monotonic decreasing function in α_2, μ as shown in Figs. 7a, 7d. But the steady state amplitude is monotonic increasing function in ω, σ as shown in Figs. 7b, 7c.





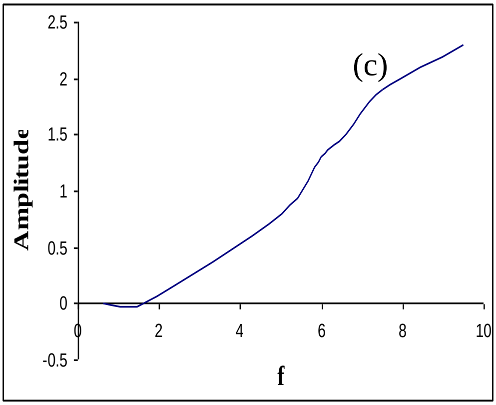


Fig.5. Effect of parameters

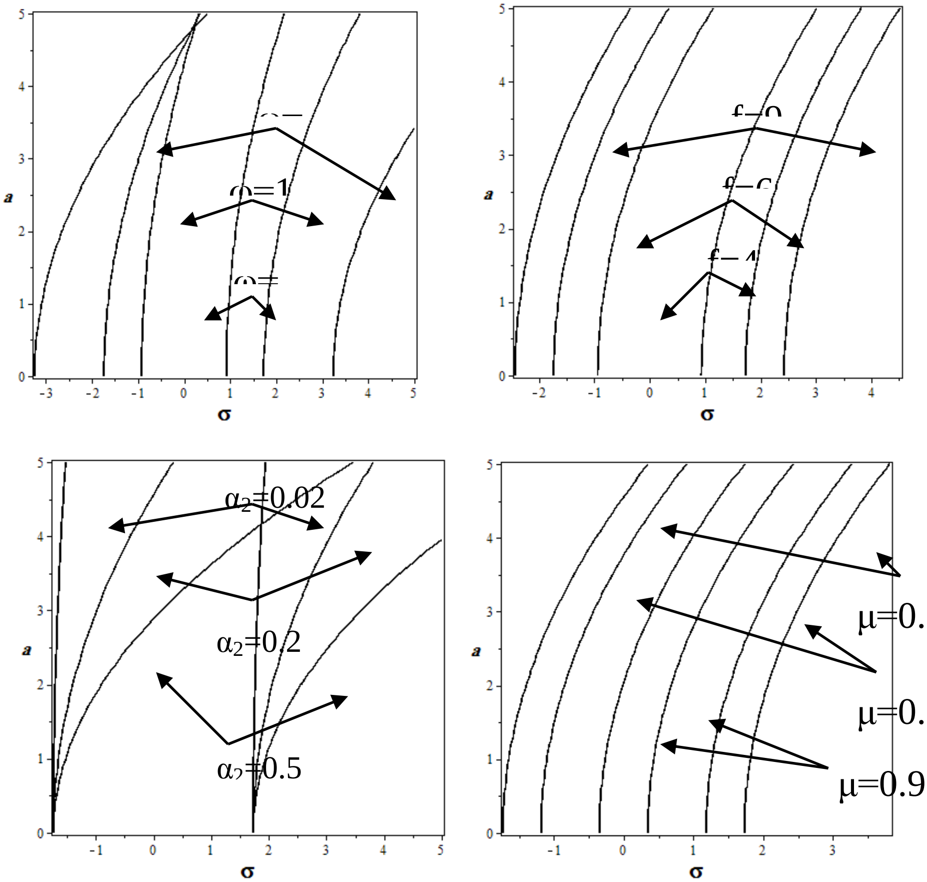


Fig.6. Frequency response curves

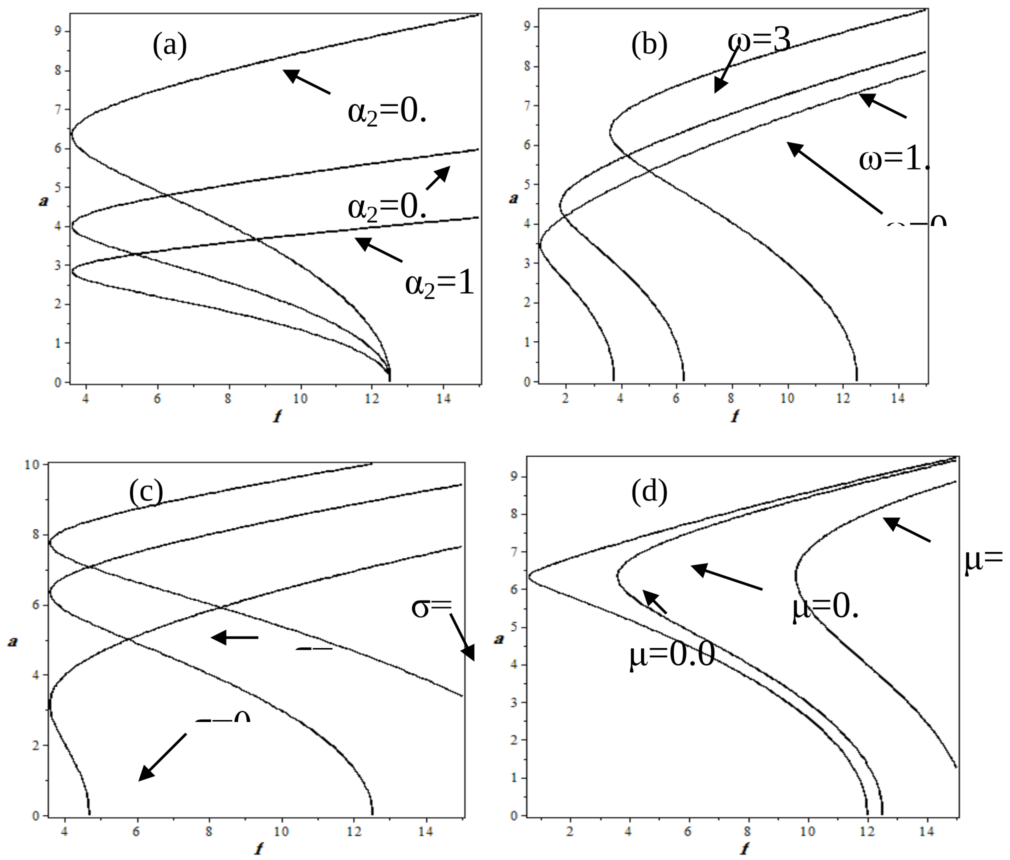


Fig.7. Excitation response curves

3. Conclusions:

The nonlinear dynamical system under multi parametric excitation of vibrations of the composite laminated rectangular plate are investigated and studied. The system is described by nonlinear differential equation a one-degree-of-freedom system that subjected to parametric excitation force. The multiple scale perturbation technique (MSPT) is applied to study the approximate solution of the given system. The stability of the system is investigated near the resonance case applying the frequency-response equations. The effects of parameter's on the main system behavior are studied numerically. From the above study the results may be concluded.

1. The worst resonance case is the sub-harmonic resonance case $\Omega = 2\omega$, the amplitude is increased to about 2.7 times of the basic case.

2. The steady state amplitude is monotonic decreasing functions to the damping coefficient μ, and the nonlinear coefficient α_2, and the natural frequency ω.

3. The steady state amplitude is monotonic increasing in the excitation force amplitude f .

4. The system stability near the resonance case applying the frequency-response equations.

5. In the linear solution if μ is positive then the solutions are stable.

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