The relationship between intuitionistic fuzzy normal subhypergroups based on intuitionistic fuzzy space and Atanassove's approach

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Abstract

The aim of this paper is to present a formulation of intoitionstic fuzzy normal sub-hypergroup (intuitionistic fuzzy normal Hv -subgroup) in intuitionistic fuzzy hyper- group based on the notion of intuitionistic fuzzy space as a direct generalization of fuzzy normal Hv -subgroup based on the notion of fuzzy universal set. A relation between intuit-tionistic fuzzy normal sub-hypergroup (intuitionistic fuzzy normal Hv -subgroup) based on intuition- istic fuzzy space and fuzzy normal sub-

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hypergroup based on fuzzy space is obtained in terms of induction and correspondence and the relation between the intuitionistic fuzzy normal subhyper group (intuitionistic fuzzy normal Hv- subgroup) and the classical intuitionistic fuzzy normal sub-hypergroup in the sense of Davvaz is obtained.

Keywords: *Intuitionistic fuzzy space,Intuitionistic fuzzy hypergroup, Intuitionistic fuzzy subhypergroup, Intuitionistic fuzzy normal subgroup.*

1. Introduction and basic concepts

The concept of hyperstructure has been introduced in 1934 by the French mathematician F. Marty [26] at the 8th Congress of Scandinavian Mathematicians, in classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstruc- ture, the composition of two elements is a set. Formally, if $\boldsymbol{H}$ is a nonempty set and $P^{*}(H)$ is the set of all nonempty subsets of $\boldsymbol{H}$ then we consider maps of the following type:

$$f_i : H \times H \to P^{*}(H)$$

where $i = 1, 2, \ldots, n$ and n is a positive integer. The maps f_i are called (binary) hyperoperations. A hypergroup H is a hyperstructure (H, $\circ$) satisfying

(1) $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$,

(2) $x \circ H = H \circ x = H$ for all $x \in H$.

If (H, $\circ$) satisfies only condition (1) then (H, $\circ$) is called a semihypergroup and if we replace condition (1) by $x \circ (y \circ z) \cap (x \circ y) \circ z = \varphi$ (which is obviously weaker than condition (1)), then the hyperstructure (H, $\circ$) is called an H_V -group. The concept of fuzzy set has been introduced by Zadeh [33] in 1965. Fuzzy set theory generalizes classical set theory in a way that membership degree of an object to a set is not restricted to the integers 0 and 1, but may take on any value in [0, 1]. Therefore a fuzzy subset ***A*** of a set ***X*** is

a function $A : X \to [0, 1]$, usually this function is referred to as the membership function and denoted by $\mu_A(x)$. Some mathematicians use the notation $\mathbf{A}(x)$ to denote the membership function instead of $\mu_A(x)$. A fuzzy subset ***A*** is written symbolically in the form $A = \{(x, \mu_A(x)) : x \in X\}$. Since the introduction of the notion of fuzzy sets many researches were conducted to generalize and carry out ordinary mathematical concepts to fuzzy mathematical concepts. From that time, the theory of fuzzy mathematics has been developed in applications in in a wide variety of fields. Atanassov [8], [9] generalized the notion of fuzzy sets to new operations defined over the intuitionistic fuzzy sets and introduced theory and applications on the intuitionistic fuzzy sets. . An intuitionistic fuzzy set A of a universe of discourse ***X*** is an object having the form: $\boldsymbol{A} = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where the functions $\mu_A, \nu_A : X \to [0, 1]$ define the degree of membership and nonmembership respectively of the element $x \in X$ such that $0 \leq \mu_A + \nu_A \leq 1$. Obviously every fuzzy set has the form $\{(x, \mu_A(x), 1 - \mu_A(x)) : x \in X\}$. Therefore if it happens that $\mu_A(x) = 1 - \nu_A(x)$ for all $\boldsymbol{x} \in \boldsymbol{X}$ then the intuitionistic fuzzy set ***A*** will be a fuzzy set. Since then, a great number of practical results appeared in the area of Atanassov's intuitionistic fuzzy sets. Davvaz [12], [13] introduced a brief survey of the theory of H_V -structures and isomor- phism theorems of polygroups. Davvaz et al, [10], [11], [14] introduced the notions of fuzzy as H_V -groups, fuzzy homo- morphisms and roughness based on fuzzy ideals In [15], [16], Davvaz et al. introduced the concept of fuzzy hyperalgebras, which is a direct extension of fuzzy algebras (fuzzy sub-hypergroup, fuzzy hyperring, etc.)

Let $(H, \circ)$ be a hypergroup (respectively, H_V -group) and let μ be a fuzzy subset of H. Then μ is said to be a fuzzy sub-hypergroup (respectively, fuzzy H_V -subgroup) of H if the following axioms hold:

1. $\min\{\mu(x), \mu(y)\} \leq \inf\{\mu(z) \ : \ z \in x \circ y\}$ for all $x, y \in H$,

2. $\min\{\mu(a), \mu(x)\} \leq \mu(y)$, for all $x, a \in H$ there exists $y \in H$ such that $x \in a \circ y$,

3. $\min\{\mu(a), \mu(x)\} \leq \mu(z)$. for all $x, a \in H$ there exists $z \in H$ such that $x \in z \circ a$.

This approach can be extended to fuzzy hypergroups, fuzzy hyperring and fuzzy hyperideal by using the concept of fuzzy universal sets and fuzzy hyperoperations. Rosenfeld [29] introduced the study of fuzzy algebraic structures with the concept of fuzzy groups. Dib in [20] remarked the absence of the fuzzy universal set and discussed some problems in Rosenfeld's approach [29]. Davvaz [5], [12], and Vougiouklis in [32] surveyed some classes of hypergroupsthe theory of H_V-structures. A new approach to define and study fuzzy hypergroups and fuzzy hyperrings is given by Davvaz et al.[15], [16], which depends on the concept of fuzzy space which serves as the concept of the universal set in the ordinary algebra. This approach can be considered as a generalization and a new formulation of Dib's approach of the fuzzy group and fuzzy subgroup which depends on the notion of fuzzy space. He introduced the

concept of fuzzy space to replace the concept of universal set in the ordinary case. Dib and Hassan [21] defined the fuzzy normal subgroup in a fuzzy group by using the concept of fuzzy spaces.
The notion of intuitionistic fuzzy sets was introduced by Atanassov [7]. The main prob- lem in intuitionistic fuzzy mathematics is how to carry out the ordinary fuzzy concepts to the intuitionistic fuzzy case.

Davvaz in [6] introduced the relationship between intuitionistic fuzzy sets and genetics, and Mohammed Fathi and Abdul Razak Salleh [23],[24], introduced a new formulation of intuitionistic fuzzy groups by introducing the concepts of intuitionistic fuzzy space and intu- itionistic fuzzy function. The study of intuitionistic fuzzy hyperstructures is an interesting research topic of intuitionistic fuzzy sets.

Abdulmula and Salleh [1] introduced the concept of intuitionistic fuzzy hypergroup based on intuitionistic fuzzy space and Abdulmula and Salleh, [1], [2], [3] introduced the definition of the notion of intuitionistic fuzzy hyperhomomorphism of intuitionistic fuzzy hypergroups and intuitionistic fuzzy normal sub-hypergroups based on intuitionistic fuzzy space as a generalisation of fuzzy hypergroup of Davvaz et al. [10] based on fuzzy space of Dib [20]. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. Davvaz, Fathi and Salleh [15],[16] applied the concept of fuzzy sets to the theory of al- gebraic hyperstructures and defined fuzzy sub-hypergroup (respectively H_V -subgroup) of a hypergroup (respectively H_V -group) and fuzzy sub-hyperrings (respectively H_V -subrings) of a hyperrings (respectively H_V -rings) which is a generalization of the concept of Rosenfeld's fuzzy subgroup of a group. Davvaz et al. in [17], [19], [28] introduced the concept of intu- itionistic fuzzy hyperrings (rigs) based on intuitionistic fuzzy universal set by Atanassove.

Dib in [20] gave us a new approach of the notion of fuzzy normal subgroups to continue the theory of intuitionistic fuzzy subgroups obtained by Fathi[27]. After that, the intuitionistic fuzzy normal subhypergroups were introduced by Abdulmula and Salleh [2],[3], in a similar manner to Salleh [4], which depends on the concept of fuzzy space which serves as the concept of the universal set in the ordinary algebra.

We recall some definitions and results which will use it in the paper. Throughout the paper, we shall adopt the notations:

X : for a non-empty set,

I : for the closed interval $[0, 1]$ of real numbers.

L: for the lattice $\mathbf{I} \times \mathbf{I}$ with partial order defined by

(i) $(r_1, r_2) \leq (s_1, s_2)$ if and only if $r_1 \leq s_1$ and $r_2 \leq s_2$ whenever $s_1 = 0$ and $s_2 = 0$,

(ii) $(s_1, s_2) = (0, 0)$ whenever $s_1 = 0$ or $s_2 = 0$.

The concept of fuzzy space and fuzzy subspace was introduced and discussed by Dib, which play the role of universal set in ordinary classical mathematics. concept of the fuzzy space $(X, \mathbf{I})$ is the set of all ordered pairs $(x, \mathbf{I})$; $x \in X$; i.e.,

$(X, \mathbf{I}) = \{(x, \mathbf{I}) : x \in X\}$, where $(x, \mathbf{I}) = \{(x, r) : r \in \mathbf{I}\}$.

The ordered pair $(x, \mathbf{I})$ is called a fuzzy element in the fuzzy space $(X, \mathbf{I})$. A fuzzy subspace U of the fuzzy space $(X, \mathbf{I})$ is the collection of all ordered pairs (x, u_x), where $x \in U_\circ$ for some $U_\circ \in X$ and u_x is a subset of $\mathbf{I}$, which contains at least one element beside the zero element. If it happens that $x \notin U_\circ$, then $u_x = 0$. An empty fuzzy subspace is defined as $\{(x, \emptyset_x) : x \in U_\circ\}$. Let $U = \{(x, u_x) : x \in U_\circ\}$ and $V = \{(x, v_x) : x \in V_\circ\}$ be fuzzy subspaces of (X, I). The union and intersection of U and V are defined respectively as follows:

$$U \cup V = \{(x, u_x \cup v_x) : x \in U_\circ \cup V_\circ\} \qquad (1.1)$$

$$U \cap V = \{(x, u_x \cap v_x) : x \in U_\circ \cap V_\circ\} \qquad (1.2)$$

Dib et al. [22] introduced some notations as, fuzzy function $\underline{F} = (F, f_{xy}) : X \times Y \longrightarrow Z$ is said to be uniform if its comembership functions f_{xy} are identical, i.e., $f_{xy}(r, s) = f(r, s)$ for all $(x, y) \in X \times Y$, and they defined fuzzy binary operation $\underline{F} = (F, f_{xy})$ on the fuzzy space (X, I) is a fuzzy function $\underline{F}$ from $X \times X$ to X with comembership functions $f_{xy} : I \times I \longrightarrow I$ that satisfy

1. $f_{xy}(r, s) = 0$ iff $r = 0$ or $s = 0$,
2. f_{xy} are onto , i.e, $f_{xy}(I \times' I) = I$ for all $(x, y) \in X \times X$.

Abdul Razak Salleh [4] introduced properties fuzzy homomorphisms of fuzzy groups Fuzzy normal subgroups were studied by Liu [31] is one among them. Dib and Hasssan [21] used the concept of fuzzy spaces to define the fuzzy normal subgroup in a fuzzy group, If $(U, \underline{F})$ where $U = \{(z, u_z) : z \in U_\circ\}$ is a fuzzy sub-hypergroup of the fuzzy hypergroup $((H, I), \underline{F})$ then for every fuzzy element (x, I) of (H, I) the fuzzy subspace defined by $(x, I)U = (x, I)\underline{F}\, U = \{(xF z, f_{xz}(I, u_z))\}$, is called a left coset of the of fuzzy sub-hypergroup $(U, \underline{F})$. a right coset of the of fuzzy sub-hypergroup $(U, \underline{F})$ is defined by the fuzzy subspace

$$U(x, I) = U\underline{F}(x, I) = \{(zFx, f_{zx}(u_z, I))\}.$$

A fuzzy sub-hypergroup U of the fuzzy hypergroup $((H, I), \underline{F})$ is called fuzzy normal sub- hypergroup if

(1) U is associative in $((H, I), \underline{F})$.

(2) $(x, I)U = U(x, I)$ such that $x \in H$.

In this paper, we define intuitionistic fuzzy normal H_V -subgroup to a given intuitionistic fuzzy hypergroup through the concept of intuitionistic fuzzy space introduced by [23]. Also, we study the relationship between the introduced intuitionistic fuzzy normal sub-hypergroup (intuitionistic fuzzy normal H_V -subgroup) and the classical ones by Davvaz is established.

The structure of this paper is as follows: We introduce the concept of intuitionistic fuzzy normal sub-hypergroup (intuitionistic fuzzy normal H_V -subgroup) in intuitionistic fuzzy hy- pergroup based on the concept of intuitionistic fuzzy space as a direct generalization of the concept of fuzzy normal group in fuzzy groups, through the new approach of fuzzy space. A relationship between the introduced intuitionistic fuzzy normal sub-hypergroup (intuitionistic fuzzy normal H_V -subgroup) and the classical ones by Davvaz is established.

2 . Hypergroups and fuzzy hypergroups

In this section, we summarize the preliminary definitions and results required in the sequel. Let $\boldsymbol{H}$ be a non-empty set and let $P^*(H)$ be the set of all non-empty subsets of $\boldsymbol{H}$. A hyperoperation on $\boldsymbol{H}$ is a map $\circ : H \times H \longrightarrow P^*(H)$ and the couple $(H, \circ)$ is called a hypergroupoid. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are non-empty subsets of $\boldsymbol{H}$, then we denote

$$A \circ B = \cup_{a \in A, b \in B} a \circ b, x \circ A = \{x\} \circ A \text{ and } A \circ x = A \circ \{x\}.$$

A hypergroupoid (H, ∘) is called a semihypergroup if for all of x, y, z of H we have (x ∘y) ∘z = x ∘ (y ∘ z), which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$$

We say that a semihypergroup (H, ∘) is a hypergroup if for all x ∈ H , we have x ∘H = H ∘x.

A hypergroupoid (H, ∘) is an H_v -group, if all x, y, z ∈ H, the following conditions hold:

• (x ∘ y) ∘ z ∩ x ∘ (y ∘ z) = ∅

• x ∘ H = H ∘ x = H.

We look at some examples of H_v –groups [32].

(1) Let (G, ·) be a group and R be an equivalence relation on G. In G/R consider the hyperoperation defined by x y = {z| z ∈ x · y}, where x denotes the equivalence class of the element ***x***. Then (G, .) is an Hv -group which is not always a hypergroup.

(2) On the set $\mathbb{Z}_{mn}$ consider the hyperoperation $\oplus$ defined by setting $0 \oplus m = \{0, m\}$ and $x \oplus y = x + y$ for all $(x, y) \in \mathbb{Z}_{mn}^2 - \{(0, m)\}$. Then $(\mathbb{Z}_{mn}, \oplus)$ is an H_v-group. $\oplus$ is weak associative but not associative.

Davvaz introduced the notions of fuzzy subhypergroup, fuzzy hyperideal and fuzzy sub- hypermodule using Rosenfeld's approach. That is he assumed a given ordinary hypergroup, hyperring and hypermodule and defined a fuzzy subhypergroup, hyperideals and hypermodule. Here, we recall the following definition from [12].

Definition 2.1 ([12]) Let (H, .) be a hypergroup (or Hv -group) and let ***μ*** be a fuzzy subset of ***H***. Then ***μ*** is said to be a fuzzy subhypergroup (or fuzzy Hv -subgroup) of ***H*** if the following axioms hold:

1. min {μ(x), μ(y)} ≤ inf α∈x.y {μ(α)}, ∀ *x, y* ∈ ***H*** ,

2. for all x, a $\in$ ***H*** there exists ***y*** $\in$ ***H*** such that ***x*** $\in$ ***a.y*** and min{μ(a), μ(x)} ≤ μ(y).

Definition 2.2 ([12]) Let ***H*** be a hyperring and let ***μ*** be a fuzzy subset of ***H***. Then *μ* is said to be a left (respectively, right) fuzzy hyperideal of ***H*** if the following axioms hold:

1. $\min\{\mu(x), \mu(y)\} \leq \inf_{\alpha \in x+y} \{\mu(\alpha)\}, \forall\ x, y \in H$,
2. for all $x, a \in H$ there exists $y \in H$ such that $x \in a + y$ and $\min\{\mu(a), \mu(x)\} \leq \mu(y)$.
3. $\mu(y) \leq \inf_{\alpha \in x.y} \{\mu(\alpha)\}$, (respectively $\mu(x) \leq \inf_{\alpha \in x.y} \{\mu(\alpha)\}$) for all $x, y \in H$,
4. for all $x, a \in H$ there exists $z \in H$ such that $x \in a.z$ and $\min\{\mu(a), \mu(x)\} \leq \mu(z)$.

In [15], Davvaz et al. defined and investigated the concept of fuzzy hypergroup, which depends on the concept of a universal set,

Definition 2.3 *Let $H^{\times I}$ be a non-empty fuzzy space. A* fuzzy hyperstructure (hypergroupoid), *denoted by $\langle H^{\times I}, \lozenge\rangle$ is a fuzzy space together with a fuzzy function having onto co-membership functions (referred to as a* fuzzy hyperoperation*) $\lozenge : H^{\times I} \times H^{\times I} \to P^*(H^{\times I})$, where $P^*(H^{\times I})$ denotes the set of all nonempty fuzzy subspaces of $H^{\times I}$ and*

$$\lozenge = (\Delta, \nabla_{xy}) \textit{with } \Delta: H \times H \to P^*(H) \ \textit{ and } \ \nabla_{xy} : I \times I \to I.$$

A fuzzy hyperoperation $\lozenge = (\Delta, \nabla_{xy})$ on $H^{\times I}$ is said to be uniform *if the associated co-membership functions ∇_{xy} are identical, i.e.,$\nabla_{xy} = \nabla$ for all $x, y \in H$. A* uniform fuzzy hyperstructure *$\langle H^{\times I}, \lozenge\rangle$ is a fuzzy hyperstructure $\langle H^{\times I}, \lozenge\rangle$ with uniform fuzzy hyperoperation.*

Recall that the action of the fuzzy function $\lozenge = (\Delta, \nabla_{xy})$ on fuzzy elements of the fuzzy space $H^{\times I}$ can be symbolized as follows:

$$\{x\} \times I \lozenge \{y\} \times I(x \Delta y, \nabla_{xy}(I \square I)) = \{x, y\} \times I.$$

Theorem 1 ([24]) *To each fuzzy hyperstructure $\langle H^{\times I}, \Diamond\rangle$ there is an associated ordinary hyperstructure $\langle H, \Delta\rangle$ which is isomorphic to the fuzzy hyperstructure $\langle H^{\times I}, \Diamond\rangle$ by the correspondence $\{x\} \times I \leftrightarrow x$.*

After introducing fuzzy hyperstructure, we are able now to define the notion of a fuzzy hypergroup.

Definition 2.4 *A fuzzy hypergroup is a fuzzy hyperstrucure $\langle H^{\times I}, \Diamond\rangle$ satisfying the following axioms:*
(i) $(\{x\} \times I\Diamond\{y\} \times I)\Diamond\{z\} \times I = \{x\} \times I\Diamond(\{y\} \times I\Diamond\{z\} \times I)$, for all $\{x\} \times I, \{y\} \times I, \{z\} \times I$ in $H^{\times I}$,
(ii) $\{x\} \times I\Diamond H^{\times I} = H^{\times I}\Diamond\{x\} \times I = H^{\times I}$, for all $\{x\} \times I$ in $H^{\times I}$.

Definition 2.5 *A fuzzy H_v-group is a fuzzy hyperstructure $\langle H^{\times I}, \Diamond\rangle$ satisfying the following conditions:*
(i) $(\{x\} \times I\Diamond\{y\} \times I)\Diamond\{z\} \times I \cap \{x\} \times I\Diamond(\{y\} \times I\Diamond\{z\} \times I) \neq \phi$, for all $\{x\} \times I, \{y\} \times I, \{z\} \times I \in H^{\times I}$,
(ii) $\{x\} \times I\Diamond H^{\times I} = H^{\times I}\Diamond\{x\} \times I = H^{\times I}$, for all $\{x\} \times I \in H^{\times I}$.

Theorem 2 ([1])(1) *For each intuitionistic fuzzy hypergroup (intuitionistic fuzzy H_v-group) there are two fuzzy hypergroups (fuzzy H_v-groups) $\langle(H, I), \widetilde{F}\rangle$ and $\langle(H, I), \widetilde{F}'\rangle$ such that $\widetilde{F} = (F, \underline{f}_{xy})$ and $\widetilde{F}' = (F, 1 - \overline{f}_{xy})$ which are isomorphic to the intuitionistic fuzzy hypergroup (intuitionistic fuzzy H_v-group) $\langle(H, I, I), \underline{F}\rangle$ by the correspondence $(x, I, I) \leftrightarrow (x, I) : x \in H$.*

(2) *To each intuitionistic fuzzy hypergroup (intuitionistic fuzzy H_v-group) there is an associated ordinary hypergroup (H_v-group) $\langle H, F\rangle$ which is isomorphic to the intuitionistic fuzzy hypergroup (intuitionistic fuzzy H_v-group) $\langle(H, I, I), \underline{F}\rangle$ by the correspondence $(x, I, I) \leftrightarrow x : x \in H$.*

2.1 Intuitionistic fuzzy normal subhypergroup

In this section we introduce the notion of intuitionistic fuzzy normal subhypergroup based on associated intuitionistic fuzzy subhypergroup, as a generalisation of the fuzzy normal subhypergroup.

Let $((H, I, I), \underline{F})$ be an intuitionistic fuzzy hypergroup having the intuitionistic fuzzy subhypergroup $(S, \underline{F})$, where $S = \{(x, \underline{s}_x, s_x), x \in S\}$. Similar to the fuzzy case and on the contrary to the ordinary case, intuitionistic fuzzy elements of the intuitionistic fuzzy subhypergroup $(S, \underline{F})$ are not necessarily associative with intuitionistic fuzzy elements of the intuitionistic fuzzy hypergroup $((H, I, I), \underline{F})$. That is, $(\alpha, I, I)\underline{F}((\beta, I, I)\underline{F}(\gamma, I, I)) = ((\alpha, I, I)\underline{F}(\beta, I, I))\underline{F}(\gamma, I, I)$, for some (α, I, I), (β, I, I) and (γ, I, I) are some intuitionistic fuzzy elements of S or (H, I, I) such that one or two of these intuitionistic fuzzy elements belong to $\boldsymbol{S}$.

Example 2.6 Let $H = \{-1, 1, -i, i\}$. Define the intuitionistic fuzzy binary hyperoperation $\underline{F} = \left(F, \underline{f}_{xy}, \overline{f}_{xy}\right)$ on (H, I, I) such that $F : H \times H \to P^*(H)$ is defined as in the following table.

$$\begin{array}{c|cccc}
F & -1 & 1 & -i & i \\
\hline
-1 & \{1\} & \{-1\} & \{i\} & \{-i\} \\
1 & \{-1\} & \{1\} & \{-i\} & \{i\} \\
-i & \{i\} & \{-i\} & \{-1\} & \{1\} \\
i & \{-i\} & \{i\} & \{1\} & \{-1\}
\end{array} \tag{2.1}$$

We have $\underline{F}$ is associative on (H, I, I). Then $\langle (H, I, I), \underline{F} \rangle$ is an intuitionistic fuzzy semihypergroup, where the comembership and canonmembership functions have the following form:

$$\underline{f}_{11}(r,s) = \underline{f}_{-1-1}(r,s) \quad = \begin{cases} 2rs & \text{if } rs \leq \frac{1}{4} \\ \frac{2}{3}(1-rs) & \text{if } rs > \frac{1}{4} \end{cases} \quad ,$$

$$\overline{f}_{11}(r,s) = \overline{f}_{-1-1}(r,s) \quad = \begin{cases} \frac{2}{3}(1-rs) & \text{if } rs \leq \frac{1}{4} \\ 2-rs & \text{if } rs > \frac{1}{4} \end{cases} \quad ,$$

$$\underline{f}_{-11}(r,s) = \underline{f}_{1-1}(r,s) \quad = \begin{cases} 2rs & \text{if } rs < \frac{1}{6} \\ 1+\frac{4}{5}(rs-1) & \text{if } rs \geq \frac{1}{6} \end{cases} \quad ,$$

$$\overline{f}_{-11}(r,s) = \overline{f}_{1-1}(r,s) \quad = \begin{cases} 2rs & \text{if } rs \leq \frac{1}{6} \\ 2-rs & \text{if } rs > \frac{1}{6} \end{cases} \quad ,$$

and the other comembership and canonmembership functions are defined by the product rs.

Clearly $\langle (H, I, I), \underline{F} \rangle$ defines a nonuniform intuitionistic fuzzy hypergroup. Also the intuitionistic fuzzy subspace $S = \{(-1, [0, \frac{1}{3}], [\frac{1}{3}, 1]), (1, [0, \frac{1}{2}], [\frac{1}{2}, 1])\}$ together with the intuitionistic fuzzy binary hyperoperation $\underline{F}$ define an intuitionistic fuzzy subhypergroup of $\langle (H, I, I), \underline{F} \rangle$. Now

$$\left((1, I, I)\underline{F}(1, [0, \frac{1}{2}], [\frac{1}{2}, 1]) \right) \underline{F}(-1, [0, \frac{1}{3}], [\frac{1}{3}, 1]) \neq$$

$$(1, I, I)\underline{F} \left((1, [0, \frac{1}{2}], [\frac{1}{2}, 1])\underline{F}(-1, [0, \frac{1}{3}], [\frac{1}{3}, 1]) \right).$$

That is, intuitionistic fuzzy elements of S are not associative with intuitionistic fuzzy elements of (H, I, I).

In the following, we define an associative intuitionistic fuzzy subhypergroup of the intuitionistic fuzzy hypergroup.

Definition 2.7 An associative intuitionistic fuzzy subhypergroup $\langle S, \underline{F}\rangle$ of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F}\rangle$ is an intuitionistic fuzzy hypergroup $\langle S, \underline{F}\rangle$ of $\langle (H, I, I), \underline{F}\rangle$ in which intuitionistic fuzzy elements of S are associative with intuitionistic fuzzy elements of (H, I, I) for any arbitrary choice of intuitionistic fuzzy elements of S and (H, I, I).

Theorem 2.8 *Let $\langle (H, I), \widetilde{F}\rangle$ with $\widetilde{F} = (F, f_{xy})$ be a uniform fuzzy hypergroup, where comembership function $f(r, 1) = f(1, r) = r$. Then every fuzzy subhypergroup S of the fuzzy hypergroup $\langle (H, I), \widetilde{F}\rangle$ is an associative in fuzzy hypergroup.*

Example 2.9 Let $H = \{-1, 1, -i, i\}$. Define the intuitionistic fuzzy binary hyperoperation $\underline{F} = \left(F, \underline{f}_{xy}, \overline{f}_{xy}\right)$ on (H, I, I) such that $F : H \times H \to P^*(H)$ is defined as in Example 2.6, the comembership and cononmembership functions are given respectively for all $x, y \in H$ by

$$\underline{f}_{xy}(r, s) = r \vee s \text{ and } \overline{f}_{xy}(r, s) = r \wedge s.$$

Obviously the intuitionistic fuzzy subspace

$$S = \{(-1, [0, \frac{1}{\beta}], [\frac{1}{\beta}, 1]), (1, [0, \frac{1}{\alpha}], [\frac{1}{\alpha}, 1]), 0 < \alpha \leq \beta \leq 1\}$$

defines an associative intuitionistic fuzzy subhypergroup of $\langle (H, I, I), \underline{F}\rangle$.

Following the above definitions and examples we have the following interesting result regarding associative intuitionistic fuzzy subgroups and as a generalisation of Theorem 2.8.

Theorem 2.10 *Let $\langle (H,I,I), \underline{F}\rangle$ with $\underline{F} = \left(F, \underline{f}, \overline{f}\right)$ be a uniform intuitionistic fuzzy hypergroup. If the comembership functions $\underline{f}(r,1) = \underline{f}(1,r) = r$ and the cononmembership functions $\overline{f}(r,1) = \overline{f}(1,r) = r$, then every intuitionistic fuzzy subhypergroup of the intuitionistic fuzzy hypergroup $\langle (H,I,I), \underline{F}\rangle$ is an associative intuitionistic fuzzy subhypergroup.*

Proof 1 The proof is directly obtained from the properties of the comembership functions $\underline{f}_{xy}$ and the cononmembership functions $\overline{f}_{xy}$ as generalisation of Theorem 2.8.

Corollary 2.11 *Let $\langle (H,I,I), \underline{F}\rangle$, where $\underline{F} = \left(F, \underline{f}, \overline{f}\right)$, be a uniform intuitionistic fuzzy hypergroup. If $\underline{f}$ and $\overline{f}$ are t-norm functions, then every intuitionistic fuzzy subhypergroup of the intuitionistic fuzzy hypergroup $\langle (H,I,I), \underline{F}\rangle$ is an associative intuitionistic fuzzy subhypergroup.*

We will now define the notions of left and right coset of intuitionistic fuzzy subhypergroup before introducing the the concept of intuitionistic fuzzy normal hypergroup.

Definition 2.12 If $\langle S, \underline{F}\rangle$ where $S = \{(z, \underline{s}_z, \overline{s}_z) : z \in S_0\}$ is an intuitionistic fuzzy subhypergroup of the intuitionistic fuzzy hypergroup $\langle (H,I,I), \underline{F}\rangle$ then for every intuitionistic fuzzy element (x,I,I) of (H,I,I),
(1) the intuitionistic fuzzy subspace $\langle S, \underline{F}\rangle$ is called a left coset of the intuitionistic fuzzy subhypergroup $\langle S, \widetilde{F}\rangle$ if

$$\begin{aligned}(x,I,I)S = (x,I,I)\underline{F}S &= \{(xFz, \underline{f}_{xz}(I, \underline{s}_z), \overline{f}_{xz}(I, \overline{s}_z)), z \in S_0\}\\ &= \{(F(x,z), \underline{f}_{xz}(I, \underline{s}_z), \overline{f}_{xz}(I, \overline{s}_z)), z \in S_0\}\\ &= \{(\{w\}, \underline{f}_{xz}(I, \underline{s}_z), \overline{f}_{xz}(I, \overline{s}_z)), \{w\} \subseteq H\}\\ &\subseteq P^*((H,I,I)),\end{aligned}$$

(2) In the same manner of (1), we define right coset of the intuitionistic fuzzy subhypergroup $\langle S, \underline{F}\rangle$ as follows

$$\begin{aligned}S(x,I,I) = S\underline{F}(x,I,I) &= \{(zFx, \underline{f}_{zx}(\underline{s}_z, I), \overline{f}_{zx}(\overline{s}_z, I)), z \in S_0\}\\ &= \{(F(z,x), \underline{f}_{zx}(\underline{s}_z, I), \overline{f}_{zx}(\overline{s}_z, I)), z \in S_0\}\\ &= \{(\{w\}, \underline{f}_{zx}(\underline{s}_z, I), \overline{f}_{zx}(\overline{s}_z, I)), \{w\} \subseteq H\}\\ &\subseteq P^*((H,I,I)).\end{aligned}$$

Theorem 2.13 *For any associative intuitionistic fuzzy subhypergroup $\langle S, \underline{F}\rangle$ of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F}\rangle$ the following hold*

(1) $(x, I, I)S = (h, \underline{I}_h, \overline{I}_h)S$ *for every intuitionistic fuzzy element* $(h, \underline{I}_h, \overline{I}_h) \in (x, I, I)S$ *where* $\underline{I}_h, \overline{I}_h$ *denotes the possible comembership and crononmembership values of* h *respectively.*

(2) *There is a one-to-one correspondence between any two left (right) cosets of the intuitionistic fuzzy subhypergroup* $\langle S, \underline{F}\rangle$.

(3) *There is a one-to-one correspondence between the family of left cosets and the family of right cosets of the intuitionistic fuzzy subhypergroup* $\langle S, \underline{F}\rangle$.

(4) *Any two lift cosets (right cosets) of the intuitionistic fuzzy subhypergroup* $\langle S, \underline{F}\rangle$ *are either identical or disjoint.*

Proof 2 In the same manner of the associative fuzzy subhypergroup, we will prove this theorem as follows
(1) Let $(\{h\}, \underline{I}_h, \overline{I}_h)$ be any intuitionistic fuzzy element in $(x, I, I)S$, then $(\{h\}, \underline{I}_h, \overline{I}_h) = (x, I, I)(y, \underline{s}_y, \overline{s}_y)$ for some $y \in S_0$. If $(z, \underline{s}_z, \overline{s}_z)$ is an arbitrary element of S then

$$\begin{aligned}(x, I, I)(z, \underline{s}_z, \overline{s}_z) &= (x, I, I)\left((y, \underline{s}_y, \overline{s}_y)(y^{-1}, \underline{s}_{y^{-1}}, \overline{s}_{y^{-1}})\right)(z, \underline{s}_z, \overline{s}_z)\\ &= \left((x, I, I)(y, \underline{s}_y, \overline{s}_y)\right)\left((y^{-1}, \underline{s}_{y^{-1}}, \overline{s}_{y^{-1}})(z, \underline{s}_z, \overline{s}_z)\right)\\ &\in (h, \underline{I}_h, \overline{I}_h)S.\end{aligned}$$

(2) Let $(x, I, I)S$ and $(y, I, I)S$ be any two left cosets of the intuitionistic fuzzy hypergroup S. Then

$$\begin{aligned}(x, I, I)\ S \leftrightarrow (x, I, I)\underline{F}(z, \underline{s}_z, \overline{s}_z) &\leftrightarrow (y, I, I)\underline{F}(z, \underline{s}_z, \overline{s}_z)\\ &\leftrightarrow (y, I, I)(z, \underline{s}_z, \overline{s}_z)\\ &\leftrightarrow (y, I, I)\ S,\end{aligned}$$

Thus $(x, I, I)(z, \underline{s}_z, \overline{s}_z) \leftrightarrow (y, I, I)(z, \underline{s}_z, \overline{s}_z)$ is the required one-to-one correspondence between $(x, I, I)S$ and $(y, I, I)S$. For the right cosets we use the same arrangement.
(3) Let $\{(x, I, I)S : x \in H\}$ and $\{S(x, I, I) : x \in H\}$ denote the family of left and right cosets respectively of the intuitionistic fuzzy hypergroup S. Then the required one-to-one correspondence is defined by

$$(x, I, I)S \leftrightarrow S(x, I, I).$$

(4) Let $(x, I, I)S$ and $(y, I, I)S$ be any two intersecting left cosets of the intuitionistic fuzzy subhypergroup S. Then there exists $\alpha, \beta \in S_0$ such that

$$(x, I, I)(\alpha, \underline{s}_\alpha, \overline{s}_\alpha) = (y, I, I)(\beta, \underline{s}_\beta, \overline{s}_\beta).$$

Choose any intuitionistic fuzzy element $(x, I, I)(z, \underline{s}_z, \overline{s}_z) \in (x, I, I)S$. Then

$$\begin{aligned}
(x,I,I)(z,\underline{s}_z,\overline{s}_z) &= (x,I,I)\left((\alpha,\underline{s}_\alpha,\overline{s}_\alpha)(\alpha^{-1},\underline{s}_{\alpha^{-1}},\overline{s}_{\alpha^{-1}})\right)(z,\underline{s}_z,\overline{s}_z)\\
&= \left((x,I,I)(\alpha,\underline{s}_\alpha,\overline{s}_\alpha)\right)\left((\alpha^{-1},\underline{s}_{\alpha^{-1}},\overline{s}_{\alpha^{-1}})(z,\underline{s}_z,\overline{s}_z)\right)\\
&= \left((y,I,I)(\beta,\underline{s}_\beta,\overline{s}_\beta)\right)\left((\alpha^{-1},\underline{s}_{\alpha^{-1}},\overline{s}_{\alpha^{-1}})(z,\underline{s}_z,\overline{s}_z)\right)\\
&= (y,I,I)\left((\beta,\underline{s}_\beta,\overline{s}_\beta)(\alpha^{-1},\underline{s}_{\alpha^{-1}},\overline{s}_{\alpha^{-1}})\right)(z,\underline{s}_z,\overline{s}_z) \in (y,I,I)S.
\end{aligned}$$

That is $(x, I, I)S \subset (y, I, I)S$. Similarly we can show that $(y, I, I)S \subset (x, I, I)S$. Similarly we can show the same result for the right cosets of S which proves (4).

In the following, we have two conditions for the intuitionistic fuzzy normal subhypergroup as given in the following definition.

Definition 2.14 An intuitionistic fuzzy subhypergroup S of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$ is called an intuitionistic fuzzy normal subhypergroup if

(1) S is associative in $\langle (H, I, I), \underline{F} \rangle$.

(2) $(x, I, I)S = S(x, I, I)$ for all $x \in H$.

Based on the definition of intuitionistic fuzzy normal subhypergroup in the next theorem, we will introduce a necessary and sufficient condition for intuitionistic fuzzy normal subhypergroups.

Theorem 2.15 *An intuitionistic fuzzy subhypergroup $\langle S, \underline{F} \rangle$ where $S = \{(z, \underline{s}_z, \overline{s}_z) : z \in S_0\}$ of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$ is an intuitionistic fuzzy normal subhypergroup if and only if*

(1) $((S_0, I), \tilde{F}), \tilde{F} = (F, f_{xy})$ *is a fuzzy normal subhypergroup of the ordinary hypergroup* (H, F).

(2) $\underline{f}_{xz}(I, \underline{s}_z) = \underline{f}_{z'x}(\underline{s}_{z'}, I), \overline{f}_{xz}(I, \overline{s}_z) = \overline{f}_{z'x}(\overline{s}_{z'}, I) : xFz = z'Fx$ *for all* $x \in H$ *and* $z, z' \in S_0$.

Proof 3 Assume $S = \{(z, \underline{s}_z, \overline{s}_z) : z \in S_0\}$ is an intuitionistic fuzzy normal subhypergroup of $\langle (H, I, I), \underline{F} \rangle$. From the correspondence theorem we have $((S_0, I), \tilde{F})$ is a fuzzy normal subhypergroup of the ordinary hypergroup (H, F). Using the normality of S we have $(x, I, I)S = S(x, I, I) : x \in H$. That is,

$$\{(xFz, \underline{f}_{xz}(I, \underline{s}_z), \overline{f}_{xz}(I, \overline{s}_z)) : z \in S_0\} = \{(zFx, \underline{f}_{zx}(\underline{s}_z, I), \overline{f}_{zx}(\overline{s}_z, I)) : z \in S_0\}.$$

Therefore for every $z \in S_0$ there exists $z' \in S_0$ such that $xFz = z'Fx$. In other words $(x, I)\tilde{F}(S_0, I) = (S_0, I)\tilde{F}(x, I)$. Hence (S_0, I) is a fuzzy normal subhypergroup of the ordinary hypergroup (H, F) which proves (1). (2) follow directly from the definition.

2.2 Intuitionistic fuzzy normal subhypergroups induced by intuitionistic fuzzy subsets

Let $\langle (H, I, I), \underline{F} \rangle$ be an intuitionistic fuzzy hypergroup and let A be an intuitionistic fuzzy subset of H, such that A induces the intuitionistic fuzzy subhypergroups $H_o(A), H_{lu}(A)$ and $H_{ul}(A)$, for these intuitionistic fuzzy subspaces. It was introduced as a generalization of [20], A induces the intuitionistic fuzzy subgroups $\langle H_o(A)\underline{F}\rangle$, $\langle H_{lu}(A)\underline{F}\rangle$ and $\langle H_{ul}(A)\underline{F}\rangle$ of $\langle (H, I, I), \underline{F}\rangle$ if

(1) $\langle (A_o, I), \widetilde{F}\rangle, \widetilde{F} = (F, f_{xy})$ is a fuzzy subhypergroup of (H, F),

(2) for all $x, y \in A_o$

$$\underline{f}_{xy}\left(\underline{A}(x), \underline{A}(y)\right) = \underline{A}\left(xFy\right), \; \overline{f}_{xy}\left(\overline{A}(x), \overline{A}(y)\right) = \overline{A}\left(xFy\right),$$

For the intuitionistic fuzzy subspaces, which are induced by intuitionistic fuzzy subsets, the results of intuitionistic fuzzy subhypergroups take a special form. We will define the associativity of the intuitionistic fuzzy subspace induced by intuitionistic fuzzy subsets.

Definition 2.16 The intuitionistic fuzzy subset A is said to be associative in the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F}\rangle$ if the intuitionistic fuzzy subspace $H_{lu}(A) = \{(x, [0, \underline{A}(x)], [\overline{A}(x), 1]), \underline{A}(x) \neq 0, \overline{A}(x) \neq 1\}$ is associative in $\langle (H, I, I), \underline{F}\rangle$.

Remark 1 By saying an intuitionistic fuzzy normal subhypergroup we mean an intuitionistic fuzzy normal subhypergroup based on intuitionistic fuzzy space.

It is easy to show that if the intuitionistic fuzzy subspace $H_{lu}(A)$ is associative in $\langle (H, I, I), \underline{F}\rangle$, then $H_\circ(A)$ and $H_{ul}(A)$ are associative in $\langle (H, I, I), \underline{F}\rangle$, for intuitionistic fuzzy subspaces, which are induced by an intuitionistic fuzzy subset A.

Theorem 2.17 *Let A be an intuitionistic fuzzy subset of H, then the intuitionistic fuzzy subspace $H_{lu}(A)$, $H_{\circ}(A)$ or $H_{ul}(A)$, is an intuitionistic fuzzy normal subhypergroup in the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$ iff*

(1) $\langle (A_{\circ}, I), \widetilde{F} \rangle$, *where* $\widetilde{F} = (F, f_{xy})$ *is a fuzzy normal subhypergroup of* (H, F)

(2) *for all* $x \in H$ *and* $z, z' \in A_{\circ}$, *if* $xFz = z'Fx$

$$\underline{f}_{xy}(1, \underline{A}(z)) = \underline{f}_{\acute{z}x}(\underline{A}(z'), 1), \; \overline{f}_{xy}\left(1, \overline{A}(z)\right) = \overline{f}_{\acute{z}x}\left(\overline{A}(z'), 1\right),$$

(3) *A is associative in* $\langle (H, I, I), \underline{F} \rangle$,

(4) *for all* $x, y \in A_{\circ}$,

$$\underline{f}_{xy}(\underline{A}(x), \underline{A}(y)) = \underline{A}(xFy), \; \overline{f}_{xy}\left(\overline{A}(x), \overline{A}(y)\right) = \overline{A}(xFy).$$

Remark 2 If the ordinary hypergroup (H, F) is an abelian hypergroup, then
(i) the fuzzy hypergroup $\langle (H, I), \widetilde{F} \rangle$ is abelian fuzzy hypergroup.
(ii) the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$ is abelian intuitionistic fuzzy hypergroup.

Theorem 2.18 *If $H_{lu}(A)$ is an intuitionistic fuzzy normal subhypergroup in $\langle (H, I, I), \underline{F} \rangle$, then the family of cosets:*

$$(x, I, I)H_{lu}(A) = \{(xFy, [0, \underline{f}_{xy}(1, \underline{A}(y))], [\overline{f}_{xy}(0, \overline{A}(y)), 1]); y \in A_0\}$$

for all $x \in H$ of the intuitionistic fuzzy subhypergroup $H_{lu}(A)$ with respect to the binary hyperoperation,

$$\begin{aligned}(x, I, I)H_{lu}(A)\underline{F}(y, I, I)H_{lu}(A) &= \left(xFy, \underline{f}_{xy}(I \times I), \overline{f}_{xy}(I \times I)\right) H_{lu}(A) \\ &= (\{z\}, I, I)\, H_{lu}(A),\end{aligned}$$

is isomorphic to the hypergroup (H, F).

Remark 3 (1) From Theorem 2, each intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$ with the intuitionistic fuzzy binary hyperoperation $\tilde{F} = (F, \underline{f}_{xy}, \overline{f}_{xy})$ is isomorphic to the fuzzy hypergroup $\langle (H, I), \tilde{F} \rangle, \tilde{F} = (F, f_{xy})$. Therefore, if $\langle S, \underline{F} \rangle$ is an intuitionistic fuzzy normal subhypergroup of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$, then $\langle S, \underline{F} \rangle$ is an intuitionistic fuzzy normal subhypergroup of fuzzy hypergroup $\langle (H, I), \tilde{F} \rangle$.

(2) From Theorem 2, each intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$ with the intuitionistic fuzzy binary hyperoperation $\tilde{F} = (F, \underline{f}_{xy}, \overline{f}_{xy})$ is isomorphic to the ordinary hypergroup $\langle H, F \rangle$. Therefore, if $\langle S, \underline{F} \rangle$ is an intuitionistic fuzzy normal subhypergroup of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$, then $\langle S, \underline{F} \rangle$ is an intuitionistic fuzzy normal subhypergroup of ordinary hypergroup $\langle H, F \rangle$.

(3) By saying a classical fuzzy normal subhypergroup we mean a fuzzy normal subhypergroup based on Rosenfeld's approach.

2.3 The relationship between fuzzy normal subhyper groups and classical fuzzy normal subhypergroups

In this section, we establish relationship between intuitionistic fuzzy normal subhypergroups based on intuitionistic fuzzy space and the classical intuition- stic fuzzy normal subhypergroups as follows.

Let $\langle (H, I, I), \underline{F} \rangle$, $\underline{F} = (F, \underline{f}, f)$ be uniform intuitionistic fuzzy hypergroup. We assume that the comembership functions $\underline{f}_{xy}$ and the cononmembership functions $\overline{f}_{xy}$ are t-norm functions. If the intuitionistic fuzzy subset A induces intuitionistic fuzzy normal subhypergroups then we have,

(1) $\langle (A_o, I), \tilde{F} \rangle$ is a fuzzy normal subhypergroup of (H, F).

(2) for all $x, y \in A_o$, we have

$$\underline{A}(xFy) = \underline{f}_{xy}(\underline{A}(x), \underline{A}(y)), \ \overline{A}(xFy) = \overline{f}_{xy}(\overline{A}(x), \overline{A}(y)).$$

(3) $A(z) = A(z')$ for all $xFz = z'Fx, x \in H, z, z' \in A_o$. (1) and (2) lead A to be a classical intuitionistic fuzzy subhypergroup of (H, F) and (3) can be written as: (4) $A(z) = A(xFzFx^{-1}), x \in H, z \in A_o$, which gives,

$$A(xFzFx^{-1}) \geq A(z), x, z \in H.$$

Therefore, A is a classical intuitionistic fuzzy normal subhypergroup. We notice that (4) can, also, be written as

$$A(xFz) = A\left(x^{-l}F(xFz)Fx\right) = A\left(zFx\right),$$

which means that A is a classical intuitionistic fuzzy normal subhypergroup. Then for all intuitionistic fuzzy normal subhypergroup of the uniform intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$, $\underline{F} = (F, \underline{f}_{xy}, \overline{f}_{xy})$ where $\underline{f}_{xy}, \overline{f}_{xy}$ is a t-norm, is a classical intuitionistic fuzzy normal subhypergroup.

In the following theorem, we give the relation between the introduced intuitionistic fuzzy normal subhypergroup based on intuitionistic fuzzy space and the classical intuitionistic fuzzy normal subhypergroup.

Theorem 2.19 *If (Y, F) is an ordinary normal subhypergroup of the the ordinary hypergroup (H, F), then for each intuitionistic fuzzy subset A of H with support $A_0 = Y$. There is an intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{P} \rangle$ such that the induced intuitionistic fuzzy subspace $\mathbf{H}(A)$, is an intuitionistic fuzzy subhypergroup of $\langle (H, I, I), \underline{P} \rangle$*

Proof 4 Assume that $\langle Y, F \rangle$ is an ordinary normal subhypergroup of the ordinary hypergroup $\langle H, F \rangle$. Let A be an intuitionistic fuzzy subset of H such that $A_0 = Y$. It is clear that (A_0, F) is an ordinary normal subhypergroup of the ordinary hypergroup (H, F). Now we define an intuitionistic fuzzy binary hyperoperation on the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{P} \rangle$, where $\underline{P}_{xy}, \overline{P}_{xy}$ are given t-norms as follows:

$\underline{P} = \{P, \underline{p}_{xy}, \overline{p}_{xy}\}$, where $P = F$, and $\underline{P}_{xy}(r_1, r_2) = \underline{h}_{xy}(\underline{f}(r_1, r_2)$, $\overline{P}_{xy}(s_1, s_2) = \overline{h}_{xy}(\overline{f}(s_1, s_2)$ such that $\underline{A}y \neq 0$ for all $x \in H, y \in A_0$, then

$$\underline{h}_{xy}(k) = \begin{cases} \frac{\underline{A}(xFy)}{\underline{A}y} k, & k < \underline{A}y, \\ \left(\frac{1-\underline{A}(xFy)}{1-\underline{A}y}\right)(k - \underline{A}y) + \underline{A}(xFy), & k \geq \underline{A}y \end{cases}$$

and $\underline{h}_{xy}(k) = k$ if $x \in H, y \notin A_0$, and If $\overline{A}y \neq 1$ for all $x \in H, y \in A_0$, then

$$\overline{h}_{xy}(k) = \begin{cases} 1 + \frac{1+\overline{A}(xFy)}{\overline{A}y}(k-1), & k < \overline{A}y, \\ \left(\frac{\overline{A}(xFy)}{\overline{A}y}\right) k, & k \geq \overline{A}y \end{cases}$$

If $\overline{Ay} = 1$, then $\overline{h}_{xy}(k) = k$ for all $k \in I$.
It is clear that $\underline{p}_{xy}(r,s), \overline{p}_{xy}(r,s)$ are continuous functions on $I \times I$ for all $x, y \in H$ and $\underline{p}_{xy}(r,s) = 0$ iff $r = 0$ or $s = 0$, and $\overline{p}_{xy}(r,s) = 1$ iff $r = 1$ or $s = 1$. Then it is easy to show that $\langle (H, I, I), \underline{P} \rangle$ is an intuitionistic fuzzy hypergroup. If $\boldsymbol{H}(A) = \{H_t(A); t = 0 \text{ or } ul \text{ or } lu\}$ is the induced intuitionistic fuzzy subspace by A of the intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{P} \rangle$, then from the definition of $\underline{p}_{xy}, \overline{p}_{xy}$, we have for all $x \in H, y \in A_0$,

$$\underline{p}_{xy}(1, \underline{A}y) = \underline{h}_{xy}(\underline{A}y) = \underline{A}(xFy),$$

and

$$\overline{p}_{xy}(1, \overline{A}y) = \overline{h}_{xy}(\overline{A}y) = \overline{A}(xFy),$$

which implies $\langle \boldsymbol{H}(A), \underline{P} \rangle$ is an intuitionistic fuzzy normal subhypergroup of $\langle (H, I, I), \underline{P} \rangle$.

Corollary 2.20 *Every classical intuitionistic fuzzy normal subhypergroup A of (H, F) induced an intuitionistic fuzzy normal subhypergroup relative to some intuitionistic fuzzy hypergroup $\langle (H, I, I), \underline{F} \rangle$.*

Conclusion

In this paper, we have generalized the study initiated in [27] about intuitionistic fuzzy normal subgroup to the context of intuitionistic fuzzy normal subhypergroup based on in- tuitionistic fuzzy space. A relationship between the introduced intuitionistic fuzzy normal subhypergroup (intuitionistic fuzzy normal *Hv* -subgroup) based on intuition- nistic fuzzy space and the classical intuitionistic fuzzy normal subhyper -group by Davvaz based on intuitionistic fuzzy universal set (Atanassove's approach [7]) is established.

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