دراسة ومناقشة مسألة المرونة الحرارية المعممة في وجود فجوات تحت تأثير كل من الدوران والجاذبية الأرضية باستخدام نظريات مختلفة.

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ملخص عربي :

في هذا البحث تم دراسة تأثير الجاذبية الأرضية والدوران على مادة مرنة حرارية ومسامية ذات خاصية حرارية, حيث تم استعراض معادلات المرونة الحرارية المعممة في وجود فجوات تحت تأثير الجاذبية الأرضية والدوران على هذا الجسم في صورة خطية مع عدم وجود قوى خارجية أخرى أو مصدر حراري. وذلك باستخدام ثلاثة نظريات وهي نظرية الارتباط (CD), لورد وشولمان(L-S) ذات الزمن الاسترخائي الواحد و نظرية جرين وليندساي (G-L) ذات زمني الإسترخاء. كما تم استخدام طريقة تحليل السلوك العادي للحصول على الكميات الفيزيائية المختلفة مع رسم هذه الكميات ومقارنتها في وجود وعدم وجود كلًا من الجاذبية الأرضية والدوران.

والنتائج التي خرج بها هذا البحث توضح مدى الفرق بين النظريات محل الدراسة وبواسطة عمل مقارنات بيانية للنتائج في ضوء تلك النظريات حيث يمكن القول بأن كل القيم من المعادلات الفيزيائية تتقارب إلى الصفر ومحققة لشروط الحدية وأيضا كل الدوال تكون مستمرة وكذلك الدوران والجاذبية الأرضية لهما دور كبير في دراسة المعادلات الفيزيائية, لان السعات من الكميات تكون متغيرة (متزايدة أو متناقصة) مع تزايد قيم الدوران والجاذبية الأرضية.

كما يوضح هذا العمل إن الدوران والجاذبية الأرضية لهما دور مهم في توزيع كميات الحقل باستثناء الحرارة وأيضا يوضح تأثير سرعة المصدر الحراري الموجود في الوسط على سرعة تقدم الموجات الحرارية والموجات الميكانيكية.

ونشير إلي أن جميع النتائج التي تم الحصول عليها والرسم كانت باستخدام برنامج (Matlab R2013a).

Generalized Thermoelastic Rotating Medium with Voids and Gravitational Field: AComparison of Differen Theories.

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Abstract.
The purpose of this work is to study the rotation of thermo-elastic material with voids under the effect of gravity. The entire porous medium is rotating with a uniform angular velocity. The formulation is applied under three theories of generalized thermoelasticity: Lord-Schulman with one relaxation time, Green-Lindsay with two relaxation times, as well as the coupled theory. The applied methodology in the problem is the normal mode analysis method for solving the thermal shock problem to obtain the exact expressions for the displacement components, the stresses, the temperature distribution, and the change in the volume fraction field which have been depicted graphically by the comparison between different theories (CD, L-S, G-L) in the presence and the absence of the rotation and the gravity influence due to the porous thermoelastic materials.

Keywords: generalized thermoelasticity; Gravity; Voids; Rotation; Normal mode analysis.

1. Introduction

The generalized thermoelasticity theories have been developed with the aim of removing the paradox of infinite speed of heat propagation inherent in the classical coupled dynamical thermoelasticity theory investigated by Biot [1]. In the generalized theories, the governing equations involve thermal relaxation times and they are of hyperbolic type. The extended thermoelasticity theory by Lord and Shulman [2] which introduces one relaxation time in the thermoelastic process and the temperature-rate-dependent theory of thermoelasticity by

Green and Lindsay [3] which takes into account two relaxation times are two well established generalized theories of thermoelasticity. The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory is useful for investigating various types of geological and biological materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small porous (voids), in which the void volume is included among the kinematics variables and in the limiting case of vanish this volume it reduces to the classical theory of elasticity. Puri and Cowin [4] studied the behavior of plane waves in a linear elastic material with voids. Iesan [5] presented a linear theory for thermo-elastic material with voids. He derived the basic equations and proved the uniqueness of the solution, reciprocity relation and variation characterization of the solution in the dynamical theory. Nunziato and Cowin [6] studied a nonlinear theory of elastic materials with voids. They showed that the changes in the volume fraction cause an internal dissipation in the material and this internal dissipation leads to a relaxation property in the material. Also Cowin and Nunziato [7] developed a linear theory for elastic materials with voids for the mathematical study of the mechanical behaviour of porous solids. This linearized theory of elastic materials with voids is a generalization of classical theory of elasticity and reduces to it when the dependence on change in volume fraction and its gradient are suppressed. Domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang [8]. Dhaliwal and Wang [9] also developed a heat-flux dependent theory of thermoelasticity with voids . Cicco and Diaco [10] presented a theory of thermoelastic material with voids without energy dissipation.

The effect of gravity on the wave propagation in an elastic solid medium was first considered by Bromwich in [11] treating the force of gravity as a type of body force. Sezawa in [12] studied the dispersion of elastic waves propagated on curved surfaces. In [13] Love extended the work of Bromwich which investigated the influence of gravity on superficial waves and showed that the Rayleigh wave velocity is affected by the gravity field. Recently Othman *et al*. [14-16] and Othman and Lotfy [17] have studied many problems using the effect of the gravitational field on thermoelasticity. Sengupta and Acharya [18] have studied the effect of gravity on the propagation of waves in a thermoelastic layer. Ailawalia and Narah [19] depicted the effects of rotation and gravity in the generalized thermoelastic medium. Sengupta *et al*. [20] studied the effect of gravity on some problem of propagation of waves in an anisotropic elastic solid medium. Abd-Alla *et al*. [21-24] investigated the influence of the gravity for the different theories. Ahamed [25] investigated the Stoneley waves in non-homogeneous orthotropic granular medium under the influence of a gravity field. In seismology and geophysics, the problem of the propagation of Rayleigh waves under the effect of gravity is significantly by Love [26]. Some researchers in the past have investigated different problems of the rotating media. Othman [27-29] used the normal mode analysis to study the effect of rotation on plane waves in generalized thermoelasticity with one and two relaxation times. Schoenberg and Censor [30] studied the effect of rotation on elastic waves. Othman *et al*. [31] have studied the generalized magneto-thermovisco-elastic plane waves under the effect of rotation without energy dissipation. Othman and Sarhan [32] have discussed the effect of rotation on a fibre-reinforced thermoelastic under Green-Naghdi theory and the influence of gravity. Chand et al. [33] presented an investigation of the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half space.

The purpose of the present work is to determine the components of displacement, stresses, temperature distribution and the volume fraction field in a homogenous, isotropic, thermoelastic solid with voids in case of absence and presence of rotation and gravity. The model is illustrated in the context of three theories (CD, L-S, G-L). The normal mode analysis is used to obtain the exact expressions for physical quantities. The distributions of considered variables are represented graphically.

2. Formulation of the Problem and Basic Equation

We consider a homogeneous, isotropic, thermoelastic material with voids in the un-deformed temperatureT_0,with half space $(z \geq 0)$ the rectangular Cartesian coordinate system(x, y, z) having originated on the surface$y = 0$.For two dimensional problem we assume the dynamic displacement vector as $u_i = (u_1, 0, u_3),\ i = 1,3$.

Equation of motion

The basic governing equations of a linear thermoelastic rotation medium with voids gravitational field under three theories are:

The strain-stress relation written as:

$$e_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}), \quad i, j = 1,3.$$

$$\sigma_{ij} = [\lambda e_{kk} + b\phi - \beta(1 + \upsilon \frac{\partial}{\partial t})T]\delta_{ij} + \mu e_{ij},$$

Since the medium is rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega \mathbf{n}$ where $\mathbf{n}$ is a unit vector representing the direction of the axis of the rotation, the equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [12]): centripetal acceleration $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})$ due to time varying motion only and Corioli's acceleration $2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}$, then the equation of motion in a rotating frame of reference is

$$\sigma_{ij,j} = \rho[\ddot{u}_i + \{\boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{u}\}_i + 2(\boldsymbol{\Omega} \times \dot{\boldsymbol{u}})_i\ ], \quad i, j = 1,3. \quad (3)$$

The dynamical equations of an elastic medium are given by

$$\mu\,\nabla^2 u_1 + (\lambda+\mu)\frac{\partial e}{\partial x} + b\,\frac{\partial \phi}{\partial x} - \beta(1+\upsilon\frac{\partial}{\partial t})\nabla\frac{\partial T}{\partial x} + \rho\,\mathrm{g}\frac{\partial u_3}{\partial x} = \rho[\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega\frac{\partial u_3}{\partial t}],$$

(4)

$$\mu\,\nabla^2 u_3 + (\lambda+\mu)\frac{\partial e}{\partial z} + b\,\frac{\partial \phi}{\partial z} - \beta(1+\upsilon\frac{\partial}{\partial t})\nabla\frac{\partial T}{\partial z} - \rho\,\mathrm{g}\frac{\partial u_1}{\partial x} = \rho[\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega\frac{\partial u_1}{\partial t}],$$

(5)

The equation of voids is

$$\alpha\,\nabla^2\phi - b\,e - \xi\phi - \omega_0 b\,\frac{\partial\phi}{\partial t} + m\,(1+\upsilon\frac{\partial}{\partial t})T = \rho\,\chi\frac{\partial^2\phi}{\partial t^2},$$

The heat conduction equation,

$$K\,\nabla^2 T = \rho C_E\,(1+\tau_0\,\frac{\partial}{\partial t})\frac{\partial T}{\partial t} + \beta T_0(1+n_0\tau_0\,\frac{\partial}{\partial t})\frac{\partial e}{\partial t} + mT_0(1+n_0\tau_0\,\frac{\partial}{\partial t})\frac{\partial\phi}{\partial t}.$$

The components of stress tensor are

$$\sigma_{xx} = \lambda(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}) + 2\mu\frac{\partial u_1}{\partial x} + b\phi - \beta(1+\upsilon\frac{\partial}{\partial t})T,$$

$$\sigma_{xz} = \lambda\,(\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}).$$

Where σ_{ij} are the components of stress tensor, e_{ij} are the components of strain, $\lambda,\ \mu$ are the Lame' constants, $\beta = (3\lambda + 2\mu)\alpha_t$ such that α_t is the coefficient of thermal expansion, δ_{ij} is the Kronecker delta, $i\,,\,j = x\,,z\,.$ $\alpha, b, \xi, \omega_0, m, \chi$ are the material constants due to the presence of voids, ρ is the density, C_E is the specific heat at constant strain, n_1, n_0 are parameters, $\tau_0,\ \upsilon$ are the thermal relaxation times, K is the thermal conductivity, T_0 is the reference temperature is chosen so that $|(T - T_0)/T_0| = 1$, ϕ is the change in the volume fraction field.

Eqs. (3) and (7) are the field equations of the generalized linear magneto-thermo-elasticity for a rotating media, applicable to the coupled theory, four generalizations, as follows:

1. The coupled (CD) theory, when

$$n^* = n_1 = 1, \quad \tau_0 = \upsilon_0 = 0.$$

2. Lord-Shulman (LS) theory, when
$n^* = n_1 = n_0 = 1, \quad t_1 = \upsilon_0 = 0, \ \tau_0 > 0.$
3. Green-Lindsay (G-L) theory, when
$n^* = n_1 = 1, \quad n_0 = 0, \quad t_1 = 0, \ \upsilon_0 \geq \tau_0 > 0.$
Our aim is to investigate the effect of temperature dependence of modulus of elasticity, keeping the other elastic and thermal parameters as constant. Therefore, we may assume that

$$\lambda = \lambda_0 f(T), \ \mu = \mu_0 f(T), \ \beta = \beta_0 f(T), \ \alpha = \alpha_0 f(T), \ \xi = \xi_0 f(T), \ \chi = \chi_0 f(T),$$
$$m = m_0 f(T), \quad k = k_0 f(T), \quad \omega_0 = \omega_{10} f(T), \quad b = b_0 f(T), \quad \upsilon = \upsilon_0 f(T).$$
(11)

Where $\lambda_0, \ \mu_0, \ \beta_0, \ \alpha_0, \ \xi_0, \ \chi_0, \ m_0, \ b_0, \ \omega_{10}, \ k_0, \ \upsilon_0$ are constants, $f(T)$ is a given non-dimensional function of temperature. In case of a temperature independent modulus of elasticity, $f(T) = 1,$ such that $f(T) = (1 - \alpha^* T_0),$ where α^* is the linear temperature coefficient. For the case a modulus of elasticity, the temperature is independent when $\alpha^* = 0.$
For simplification, the following non-dimensional variables are used:

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\mu}, \ (\mathrm{u}'_1, \mathrm{u}'_3) = \frac{\omega_1^*}{c_1}(\mathrm{u}_1, u_3), \ (x', z') = \frac{\omega_1^*}{c_0}(x, z),$$
$$\phi' = \frac{\omega_1^* \chi_0}{c_1^2}\phi, \ \ T' = \frac{T}{T_0},$$
$$t' = \omega_1^* t, \ \Omega' = \frac{\Omega}{\omega_1^*}, \ c_1^2 = \frac{\lambda + 2\mu}{\rho}, \ \ \omega_1^* = \frac{\rho C_E c_1^2}{k}, \upsilon = \omega_1^* \upsilon, \ \ \tau_0' = \omega_1^* \tau_0, \ \ g' = \frac{g}{c_1 \omega_1^*},$$
(12)

From Eq. (12) in to Eqs. (4)-(7) we get

$$\nabla^2 u_1 + A_1 \frac{\partial e}{\partial x} + A_2 \frac{\partial \phi}{\partial x} - A_3 (1 + \upsilon_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial x} + A_4 \frac{\partial u_3}{\partial x} = A_5 [\frac{\partial^2 u_1}{\partial t^2} - \Omega^2 u_1 - 2\Omega \frac{\partial u_3}{\partial t}],$$
(13)

$$\nabla^2 u_3 + A_1 \frac{\partial e}{\partial z} + A_2 \frac{\partial \phi}{\partial z} - A_3 (1+\upsilon_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial z} - A_4 \frac{\partial u_1}{\partial x} = A_5 [\frac{\partial^2 u_3}{\partial t^2} - \Omega^2 u_3 - 2\Omega \frac{\partial u_1}{\partial t}],$$

(14)

$$\nabla^2 \phi - A_6 e - A_7 \phi - A_8 \frac{\partial \phi}{\partial t} + A_9 (1+\upsilon_0 \frac{\partial}{\partial t}) T = A_{10} \frac{\partial^2 \phi}{\partial t^2},$$

$$\varepsilon_1 \nabla^2 T - A_{11} (1+ n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial \phi}{\partial t} - \varepsilon_2 (1+ n_0 \tau_0 \frac{\partial}{\partial t}) \frac{\partial e}{\partial t} .= (1+\tau_0 \frac{\partial}{\partial t}) \frac{\partial T}{\partial t}$$

Also, the constitutive Eqs. (8)-(10) reduce to

$$\sigma_{xx} = A_{12} [\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}] + 2 \frac{\partial u_1}{\partial x} + A_{13} \phi - A_{14} T,$$

(17)

$$\sigma_{zz} = A_{12} [\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}] + 2 \frac{\partial u_3}{\partial z} + A_{13} \phi - A_{14} T,$$

(18)

$$\sigma_{xz} = [\frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x}].$$

(19)

Where

$$A_1 = \frac{\lambda + \mu}{\mu}, \quad A_2 = \frac{b c_1^2}{\mu \omega_1^{*2} \chi}, \quad A_3 = \frac{\beta T_0}{\mu}, \quad A_4 = \frac{\rho g c_1^2}{\mu}, \quad A_5 = \frac{\rho c_1^2}{\mu}, \quad A_6 = \frac{b \chi}{\alpha},$$

$$A_7 = \frac{\xi c_1^2}{\alpha \omega_1^{*2}}, \quad A_8 = \frac{\omega_0 c_1^2}{\alpha \omega_1^{*}}, \quad A_9 = \frac{m T_0 \chi}{\alpha}, \quad A_{10} = \frac{\rho c_1^2 \chi}{\alpha}, \quad A_{11} = \frac{m c_1^2}{\rho C_E \omega_1^{*2} \chi},$$

$$\varepsilon_1 = \frac{K \omega_1^{*}}{\rho c_1^2 C_E}, \quad \varepsilon_2 = \frac{\beta}{\rho C_E},$$

$$A_{12} = \frac{\lambda}{\mu}, \quad A_{13} = \frac{b c_1^2}{\mu \omega_1^{*2} \chi}, \quad A_{14} = \frac{\beta T_0}{\mu} (1+\upsilon \omega) .$$

We define displacement potentials R and Q which relate to displacement components u_1 and u_3 as,

$$u_1 = \frac{\partial R}{\partial x} + \frac{\partial Q}{\partial z}, \text{ and } \; u_3 = \frac{\partial R}{\partial z} - \frac{\partial Q}{\partial x},$$

$$e = \nabla^2 R, \text{ and } \; (\frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x}) = \nabla^2 Q.$$

By substituting from Eq. (21) in Eqs. (13)-(16), this yield

$$[(1+A_1)\nabla^2 - A_5(\frac{\partial^2}{\partial t^2} - \Omega^2)]R + (A_4\frac{\partial}{\partial x} - 2A_5\Omega)Q + A_2\phi - A_3(1+\upsilon_0\frac{\partial}{\partial t})T = 0,$$

(22)

$$[2A_5\Omega\frac{\partial}{\partial t} - A_4\frac{\partial}{\partial x}]R + [\nabla^2 - A_5(\frac{\partial^2}{\partial t^2} - \Omega^2)]Q = 0,$$

$$-A_6\nabla^2 R + (\nabla^2 - A_7 - A_8\frac{\partial}{\partial t} - A_{10}\frac{\partial^2}{\partial t^2})\phi + A_9(1+\upsilon_0\frac{\partial}{\partial t})T = 0,$$

$$-\varepsilon_2(\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2})\nabla^2 R - A_{11}(1+n_0\tau_0\frac{\partial}{\partial t})\frac{\partial\phi}{\partial t} + \varepsilon_1\nabla^2 T - (1+\tau_0\frac{\partial}{\partial t})\frac{\partial T}{\partial t} = 0.$$

3 Normal Mode Analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$[u_1, u_3, T, R, Q, \phi, \sigma_{ij}](x, z, t) = [u_1^*, u_3^*, T^*, R^*, Q^*, \phi^*, \sigma_{ij}^*](z)\exp[i(\omega t + a x)],$$

(26)

Where $[u_1^*, u_3^*, T^*, R^*, Q^*, \phi^*, \sigma_{ij}^*](z)$ are the amplitudes of the function, ω is the complex time constant, $i = \sqrt{-1}$ and a is the wave number in *x*-direction.

Using Eq. (26) in Eqs. (22)-(25), lead to

$$(D^2 - S_2)R^* - S_3Q^* - S_4\phi^* - S_5T^* = 0,$$

$$S_6R^* + (D^2 - S_7)Q^* = 0,$$

$$(-A_6D^2 + S_8)R^* + (D^2 - S_9)\phi^* + S_{10}T^* = 0,$$

$$S_{11}(D^2 - a^2)R^* - S_{12}\phi^* + (D^2 - S_{13})T^* = 0.$$

Using Eqs. (20) and (26) in Eqs. (17)-(19), we get

$$\sigma_{xx}^* = A_{12}(D^2 - a^2)R^* + 2iau_1^* + A_{13}\phi^* - A_{14}(1+i\upsilon_0\omega)T^*,$$

$$\sigma_{zz}^* = A_{12}(D^2 - a^2)R^* + 2Du_3^* + A_{13}\phi^* - A_{14}(1+i\upsilon_0\omega)T^*,$$

$$\sigma_{xz}^* = [Du_1^* + iau_3^*].$$

Where,

$$S_1 = 1 + A_1,\quad S_2 = \frac{S_1 a^2 - A_5(\omega^2 + \Omega^2)}{S_1},\quad S_3 = \frac{2A_5\Omega - A_4 ia}{S_1},\quad S_4 = \frac{A_2}{S_1},\quad S_5 = \frac{A_3(1 + i\upsilon_0\omega)}{S_1},$$

$$S_6 = 2A_5 i\omega\Omega - A_4 ia,\quad S_7 = a^2 - A_5(\omega^2 + \Omega^2),\quad S_8 = A_6 a^2,\quad S_9 = a^2 + A_7 + A_8 i\omega - A_{10}\omega^2,$$

$$S_{10} = A_9(1 + i\upsilon_0\omega),\quad S_{11} = \frac{-\varepsilon_2(\mathrm{i}\omega - n_0\tau_0\omega^2)}{\varepsilon_1},\quad S_{12} = \frac{A_{11}(\mathrm{i}\omega - n_0\tau_0\omega^2)}{\varepsilon_1},$$

$$S_{13} = \frac{\varepsilon_1 a^2 + \mathrm{i}\omega - \tau_0\omega^2}{\varepsilon_1},\qquad \mathrm{D} = \frac{\mathrm{d}}{\mathrm{dz}}.$$

Eliminating Q^*, ϕ^* and T^* between Eqs. (27) - (30), we get the following eighth ordinary differential equation satisfied with R^*:

$$[\mathrm{D}^8 - F_1\mathrm{D}^6 + F_2\mathrm{D}^4 - F_3\mathrm{D}^2 + F_4]R^*(\mathrm{z}) = 0.$$

Where

$$F_1 = S_{13} + S_9 + S_7 + S_2 - A_6S_4 - S_5S_{11},$$

$$\begin{aligned}F_2 = {} & S_9S_{13} + S_{10}S_{12} + S_7S_{13} + S_7S_9 + S_2S_{13} + S_2S_9 + S_2S_7 - S_6S_3 + A_6S_4S_{13} \\ & + S_4S_8 - S_4S_{10}S_{11} + A_6S_4S_7 - A_6S_5S_{12} - S_5S_{11}a^2 - S_5S_9S_{11} - S_5S_7S_{11},\end{aligned}$$

$$\begin{aligned}F_3 = {} & S_7S_9S_{13} + S_7S_{10}S_{12} + S_2S_9S_{13} + S_2S_{10}S_{12} + S_2S_7S_{13} + S_2S_7S_9 - S_3S_6S_{13} \\ & - S_3S_6S_9 - S_4S_8S_{13} - S_4S_{10}S_{11}a^2 - A_6S_4S_7S_{13} + S_4S_7S_8 - S_4S_7S_{10}S_{11} \\ & - S_5S_8S_{12} - S_5S_9S_{11}a^2 + A_6S_5S_7S_{12} - S_5S_7S_{11}a^2 - S_5S_7S_9S_{11},\end{aligned}$$

$$\begin{aligned}F_4 = {} & S_2S_7S_9S_{13} + S_2S_7S_{10}S_{12} - S_3S_6S_9S_{13} - S_3S_6S_{10}S_{12} + S_4S_7S_8S_{13} - S_4S_7S_{10}S_{11}a^2 \\ & - S_5S_7S_8S_{12} - S_5S_7S_9S_{11}a^2.\end{aligned}$$

Equation (34) can be factored as

$$[(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)(\mathrm{D}^2 - k_3^2)(\mathrm{D}^2 - k_4^2)]R^*(\mathrm{z}) = 0.$$

Where $k_n^2 (n = 1, 2, 3, 4)$ are the roots of the characteristic equation of Eq. (34).

The solution of Eq. (34) which is bounded as $z \to \infty$, is given by:

$$R^* = \sum_{n=1}^{4} M_n e^{-k_n z},$$

$$\phi^* = \sum_{n=1}^{4} L_{2n} M_n e^{-k_n z},$$

(38)

$$T^{*} = \sum_{n=1}^{4} L_{3n} M_{n} e^{-k_{n}z} .$$

(39)

Where, M_n $(n=1,2,3,4)$ are constants.

To obtain the components of the displacement vector, from (36) and (37) in (20)

$$u_1^{*} = \sum_{n=1}^{4} L_{4n} M_{n} e^{-k_{n}z} ,$$

From Eqs. (36)-(41) in (31)-(33) to obtain the components of the stresses

$$\sigma_{xx}^{*} = \sum_{n=1}^{4} L_{6n} M_{n} e^{-k_{n}z} ,$$

$$\sigma_{zz}^{*} = \sum_{n=1}^{4} L_{7n} M_{n} e^{-k_{n}z} ,$$

(43)

$$\sigma_{xz}^{*} = \sum_{n=1}^{4} L_{8n} M_{n}\, e^{-k_{n}z} .$$

(44)

Where,

$$L_{1n} = \frac{-S_6}{(k_n^2 - S_7)}, \qquad L_{2n} = \frac{S_5(A_6 k_n^2 - S_8) + S_{10}(k_n^2 - S_2 + S_3 L_{1n})}{S_5(k_n^2 - S_9) + S_4 S_{10}},$$

$$L_{3n} = \frac{S_4 L_{2n} - k_n^2 + S_2 - S_3 L_{1n}}{S_5}, \qquad L_{4n} = ia - k_n L_{1n},,$$

$$L_{5n} = -(k_n + iaL_{1n}),$$

$$L_{6n} = A_{12}(iaL_{4n} - k_n L_{5n}) + 2iaL_{4n} + A_{13} L_{2n} - A_{14} L_{3n},$$

$$L_{7n} = A_{12}(iaL_{4n} - k_n L_{5n}) - 2k_n L_{5n} + A_{13} L_{2n} - A_{14} L_{3n}, \quad L_{8n} = (-k_n L_{4n} + iaL_{5n}).$$

4 Boundary conditions

In this section, we need to consider the Boundary conditions at $z = 0$, in order to determine the parameter M_n $(n=1,2,3,4)$.

(1) The mechanical boundary condition

$$\sigma_{zz} = -P_1 e^{i(\omega t + ax)}, \qquad \sigma_{xz} = 0, \qquad \frac{\partial \phi}{\partial z} = 0.$$

(45)

(2) The thermal boundary condition that the surface of the half-space is subjected to

$$T = P_2 e^{i(\omega t + ax)}.$$

(46)

Where, P_1 is the magnitude of the applied force in of the half-space and P_2 is the applied constant temperature to the boundary. Using the expressions of the variables into the above boundary conditions (45), (46), we obtain ;

$$\sum_{n=1}^{4} L_{7n} M_n = -P_1,$$

(47)

$$\sum_{n=1}^{4} L_{8n} M_n = 0,$$

(48)

$$\sum_{n=1}^{4} -k_n L_{2n} M_n = 0,$$

(49)

$$\sum_{n=1}^{4} L_{3n} M_n = P_2.$$

(50)

Invoking boundary conditions (47)-(50) at the surface $z = 0$ of the plate, we obtain a system of four equations. After applying the inverse of matrix method, we get the values of the four constants M_n $(n = 1, 2, 3, 4)$.

6 Numerical Results and Discussion

Copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as follows:

$\lambda = 7.76 \times 10^{10}\ N.m^{-2}$, $\mu = 3.86 \times 10^{10}\ kg.m^{-1}.s^{-2}$, $K = 386\, w.m^{-1}.k^{-1}$,
$\alpha_t = 1.78 \times 10^{-5}\, k^{-1}$, $\rho = 8954\ kg.m^{-3}$, $C_E = 383.1\, J.kg^{-1}.k^{-1}$,
$T_0 = 293\, K$.
$\beta = 2.68 \times 10^{6} N/m^2 \deg$, $\omega_1^* = 3.58 \times 10^{11}/s$.

The voids parameters are
$\chi = 1.753 \times 10^{-15} m^2$, $\xi = 1.475 \times 10^{10} N/m^2$, $b = 1.13849 \times 10^{10} N/m^2$,
$\alpha = 3.688 \times 10^{-5} N$, $m = 2 \times 10^{6} N/m^2 \deg$,
$\omega_0 = 0.0787 \times 10^{-3} N/m^2 s$.

The comparisons were carried out for
$x = 0.5$, $t = 0.0$[illegible] $\omega = \zeta_0 + i\zeta_1$, $\zeta_0 = -0.6$ $\xi_1 = 2$, $p_1 = 0.1$
$p_2 = 8$, $\tau_0 = 1s$,
$\upsilon = 1.5$, $a = 1.5$, $\Omega = 0.1$, $0 \le z \le 6$.

The above numerical technique, was used for the distribution of the real parts of the displacement components u_1 and u_3, the temperature distribution T, the stress components, σ_{xx}, σ_{zz} and σ_{xz} and change in the volume fraction field ϕ with distance in three theories, for these cases

(i) with and without rotation effect are shown graphically in figures 1-7 in the case of two
different values of rotation $(\Omega = 0.1\ rad/s,\ 0)$ while $g = 9.8\ m/s^2$.

(ii) with and without the gravity properties in Figures 8-14 in the case of two different
values of $(g = 0,\ g = 9.8\ m/s^2)$ while $\Omega = 0.1\ rad/s$.

The computations were carried out for a value of time $t = 0.03$. The above numerical technique, was used for the distribution of the real part of the displacement components u_1 and u_3, the thermodynamic temperature T and the stress components σ_{xx}, σ_{zz}, σ_{xz} and change in the volume fraction field ϕ with the distance z for the problem under consideration. All the considered variables depend not only on the variables t, x and z, but also depend on the thermal relaxation times τ_0 and υ_0. The results are shown in Figs. 1-14.

Figs. 1-6 show comparisons among the considered variables in the absence and presence of the gravity effect ($g = 0$, $g = 9.8\ m/s^2$).

Fig. 1 shows that the distribution of the displacement component u_1 in case of $(g = 9.8\ m/s^2, 0)$ in the context of the three theories, we notice that the displacement component u_1 distribution is increasing with increase of a gravity for $z > 0$. It is observed that the gravity has a great effect on this physical quantity. Fig. 2 shows that the distribution of the displacement component u_3 in case of $(g = 9.8\ m/s^2, 0)$ in the context of the three theories, it noticed that the displacement component u_3 distribution is increasing with increase of a gravity for $z > 0$.

Fig. 3 explain that the distribution of temperature T begins from positive value (which is the same point) in case of $(g = 9.8\ m/s^2)$ and $g = 0$, in the context of the three theories, and we notice that the gravity has a small effect in the distribution of temperature T.

Fig. 4 depicts that the distribution of change in volume fraction field ϕ begins from negative value in case of $(g = 9.8\ m/s^2)$ and $g = 0$, in the context of the three theories, however the change in volume fraction field ϕ is increases with the increase of gravity value for $z > 0$. Fig. 5 shows that the distribution of stress component σ_{xx} begins from negative value (which is distinct values with small distances) in case of $(g = 9.8\ m/s^2)$ and $g = 0$, in the context the three theories, and we deduce that the gravity has a great effect on the stress component σ_{xx}, while the distribution of stress component σ_{xx} is decreasing with increasing of the gravity. Fig. 6 determines that the distribution of stress component σ_{zz} begins from negative value (which is the same point) in case of $(g = 9.8\ m/s^2)$ and $g = 0$, in the context of the three theories, and we notice that σ_{zz} increases with increasing of the gravity. $(g = 9.8\ m/s^2)$ and $g = 0,$ Fig. 7 depicts that the distribution of stress component σ_{xz} begins from zero (which is the same point) in case of $(g = 9.8\ m/s^2)$ and $g = 0,$ in the context of the three theories, and the distribution of stress component σ_{xz} is decreasing with increasing of the gravity, and then approaches to zero with an increase in distance z.

Figures 8-14 show the distribution of the displacements u_1 and u_3, the temperature T, the stress components $\sigma_{xx}, \sigma_{zz}, \sigma_{xz}$, in case of two different values, (for absence and presence of the rotation) and in case of the gravity $(g = 9.8\ m/s^2)$.

Fig. 8 shows that the distribution of the displacement component u_1 in case of $(\Omega = 0.1\ rad/s,\ 0)$ in the context of the three theories, we notice that the displacement component u_1 distribution is increasing with increase of a rotation for $z > 0$. It is observed that the rotation has a great effect on this physical quantity. Fig. 9 shows that the distribution of the displacement component u_3 in case of $(\Omega = 0.1\ rad/s,\ 0)$ in the context of the three theories, it noticed that the displacement component u_3 distribution is increasing with increase of a rotation for $z > 0$.

Fig.10 explain that the distribution of temperature T begins from positive value (which is the same point) in case of $\Omega = 0.1\ rad/s$, and $\Omega = 0$, in the context of the three theories, and we notice that the rotation has a small effect in the distribution of temperature T. Fig. 11 depicts that the distribution of change in volume fraction field ϕ begins from negative value in case of $\Omega = 0.1\ rad/s$, and $\Omega = 0$, in the context of the three theories, however the change in volume fraction field ϕ is increases with the increase of rotation value for $z > 0$. Fig. 12 shows that the distribution of stress component σ_{xx} begins from negative value (which is distinct values with small distances) in case of

$\Omega = 0.1\, rad/s$, and $\Omega = 0$, in the context the three theories, and we deduce that the rotation has a great effect on the stress component σ_{xx}, while the distribution of stress component σ_{xx} is decreasing with increasing of the rotation. Fig. 13 determines that the distribution of stress component σ_{zz} begins from negative value (which is the same point) in case of $\Omega = 0.1\, rad/s$, and $\Omega = 0$, in the context of the three theories, and we notice that σ_{zz} increases with increasing of the rotation. Fig. 14 depicts that the distribution of stress component σ_{xz} begins from zero (which is the same point) in case of $\Omega = 0.1\, rad/s$, and $\Omega = 0$, in the context of the three theories, and the distribution of stress component σ_{xz} is decreasing with increasing of the rotation, and then approaches to zero with an increase in distance z.

3D curves 15-18 represent the relation between the physical quantities and both components of distance, in the presence of the rotation $\Omega = 0.1\, rad/s$, and the modulus of elasticity is dependent on the gravity ($g = 9.8\, m/s^2, 0$) in the context of the (G-L) theory. These figures are very important to study the dependence of these physical quantities on the vertical component of distance. The curves obtained are highly depending on the vertical distance and all the physical quantities are moving in wave propagation.

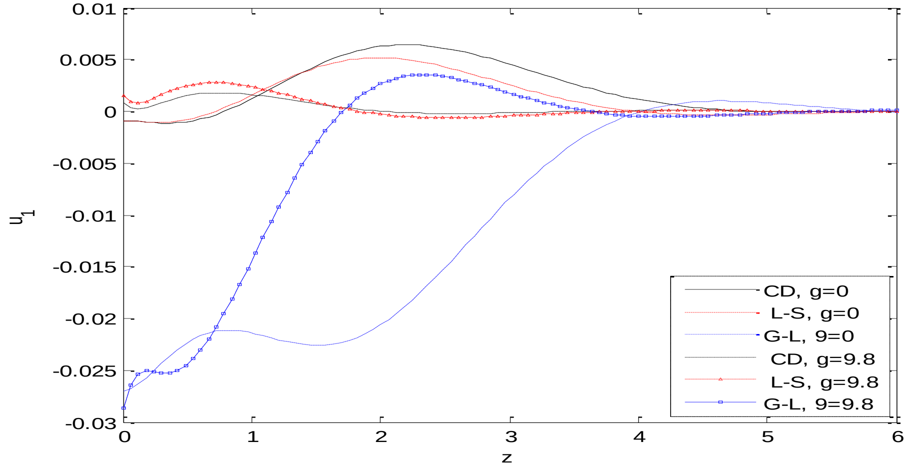

Figure 1: The displacement component u_1 distribution against z with and without gravity.

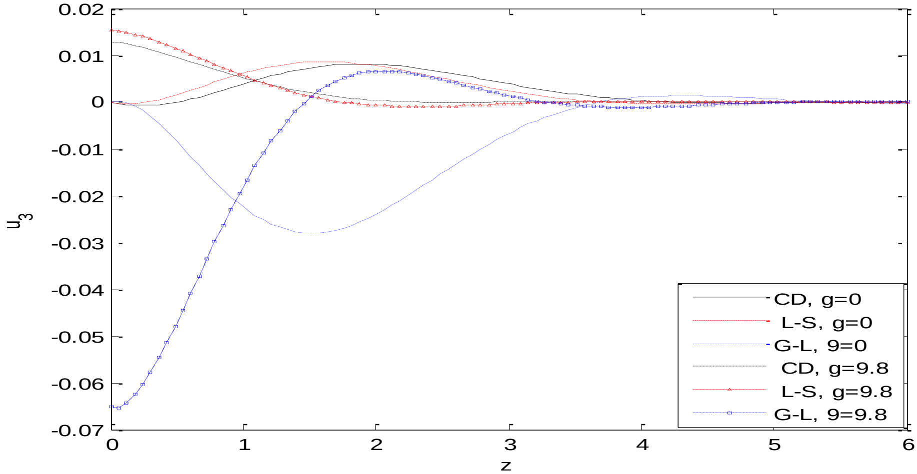

Figure 2: The displacement component u_3 distribution against z with and without gravity.

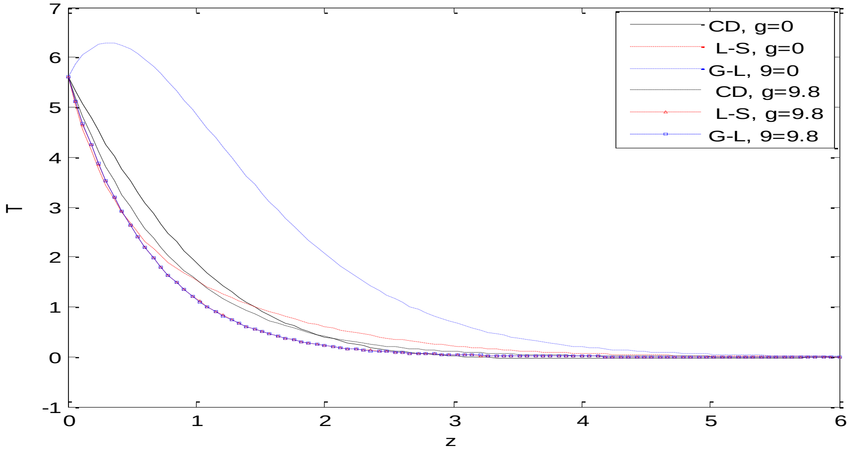

Figure 3: The displacement of the temperature T against z with and without gravity.

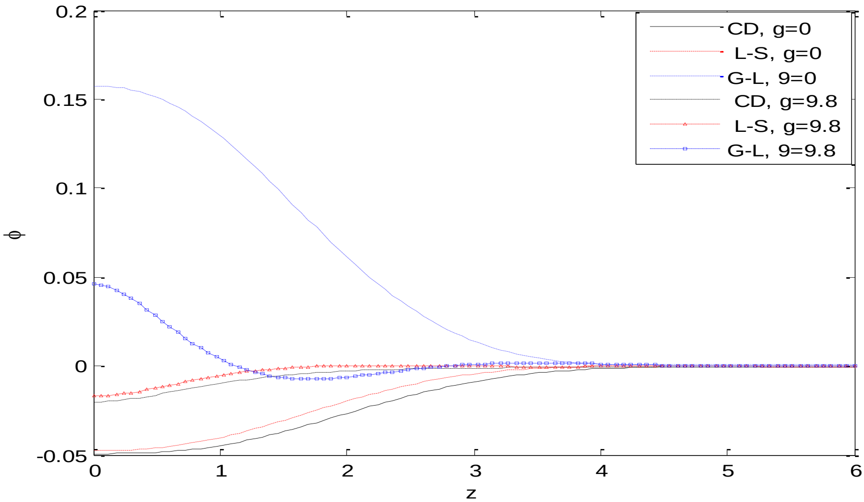

Figure 4: The displacement of volume fraction field ϕ against z with and without gravity.

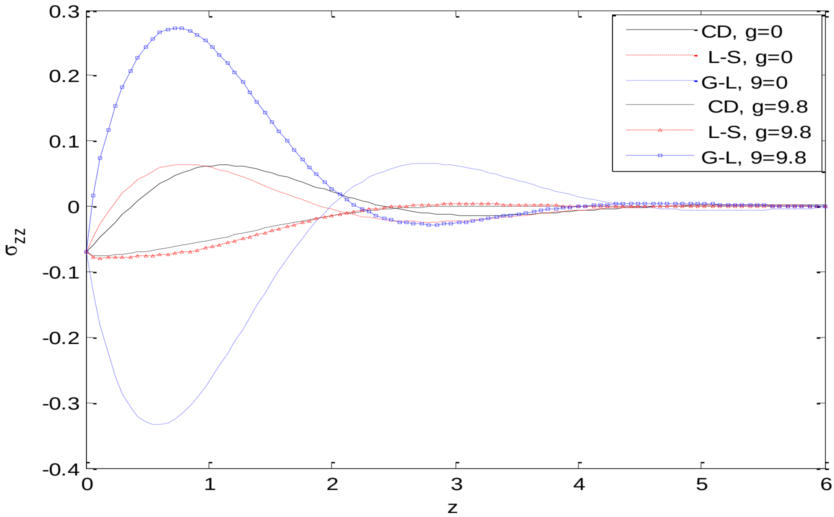

Figure 5: The displacement of the stress tensor σ_{zz} against z with and without gravity.

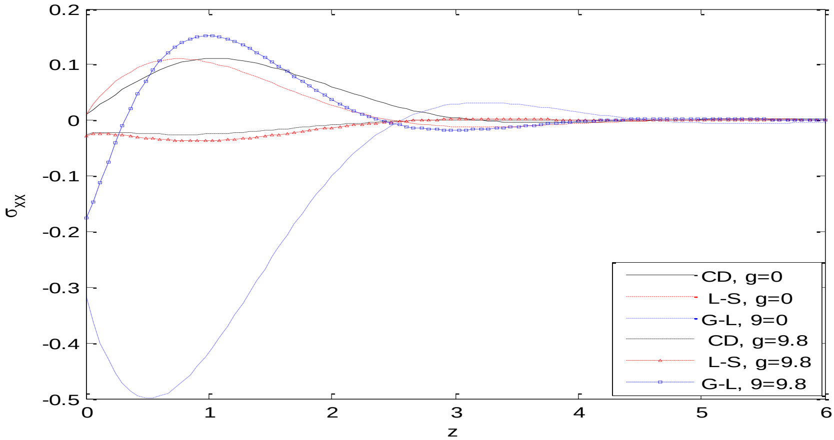

Figure 6: The displacement of the stress tensor σ_{xx} against z with and without gravity.

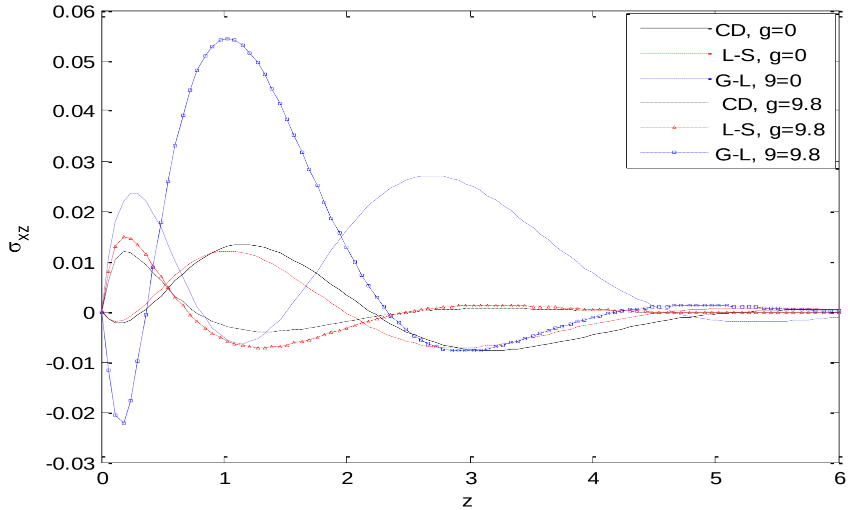

Fig. 7: The displacement of the stress tensor σ_{xz} against z with and without gravity.

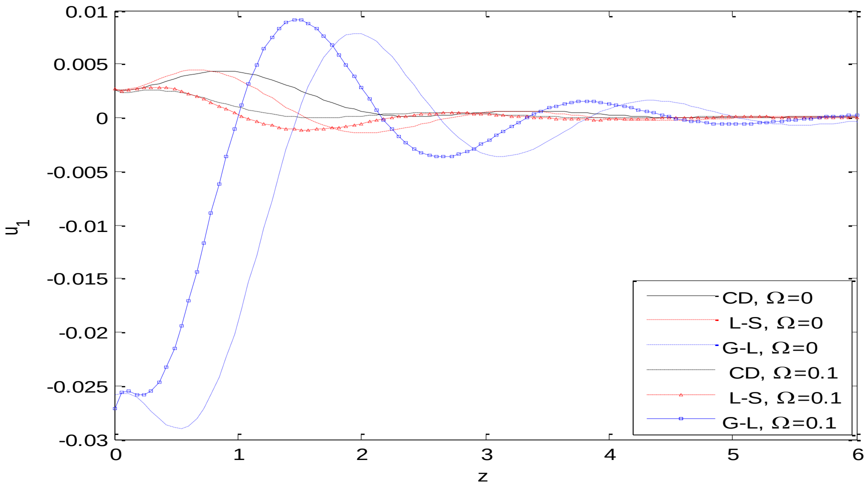

Fig. 8 Horizontal displacement distribution u_1 in the absence and presence of rotation

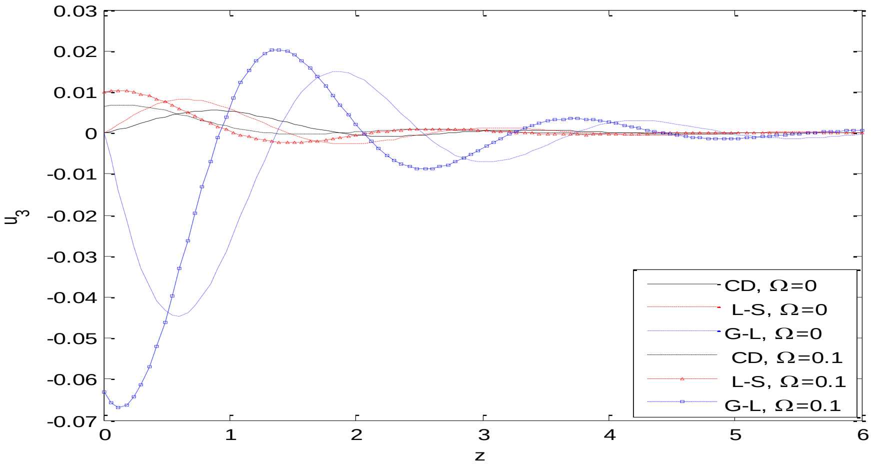

Fig. 9 Vertical displacement distribution u_3 in the absence and presence of rotation

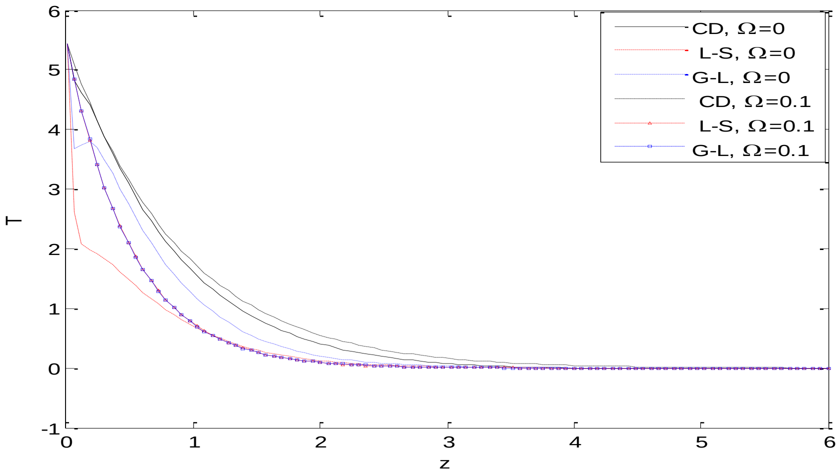

Fig. 10 Thermodynamic temperature distribution T in the absence and presence of rotation

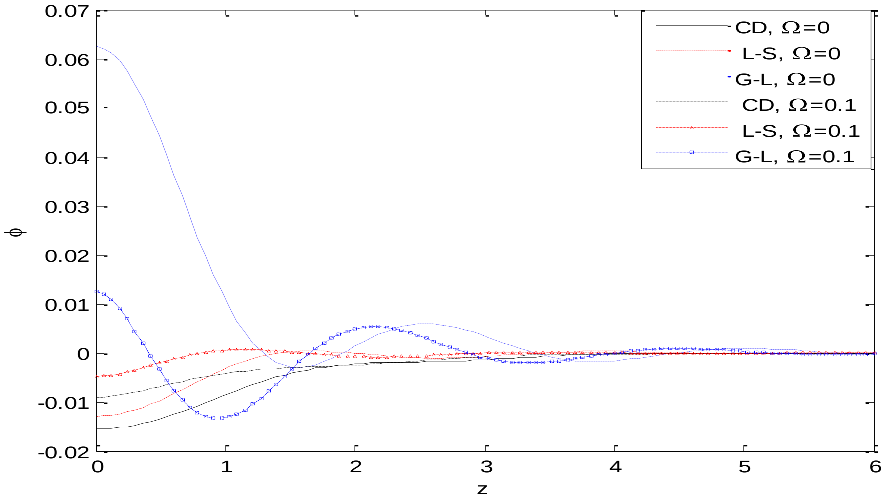

Fig.11 The displacement of volume fraction field ϕ in the absence and presence of rotation.

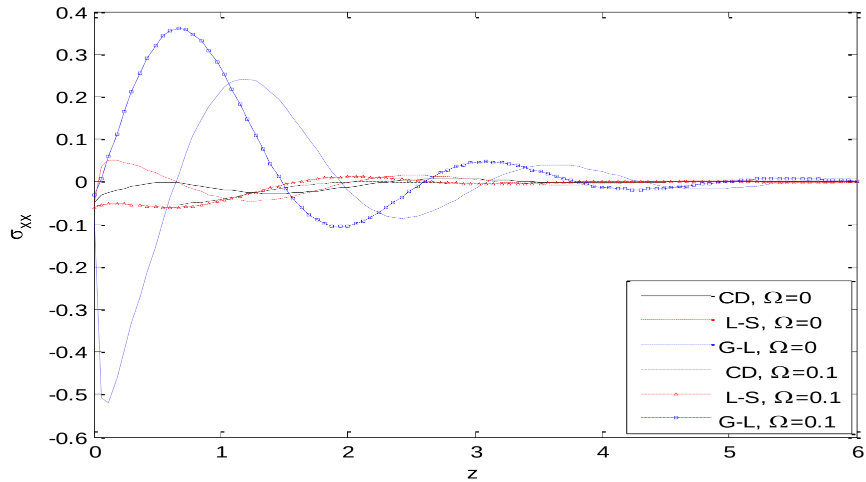

Fig.12 Distribution of stress component σ_{xx} in the absence and presence of rotation

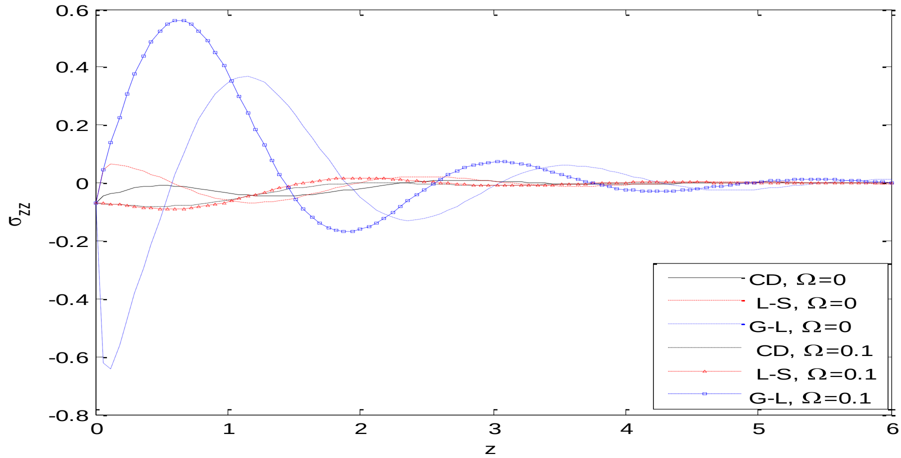

Fig. 13 Distribution of stress component σ_{zz} in the absence and presence of rotation.

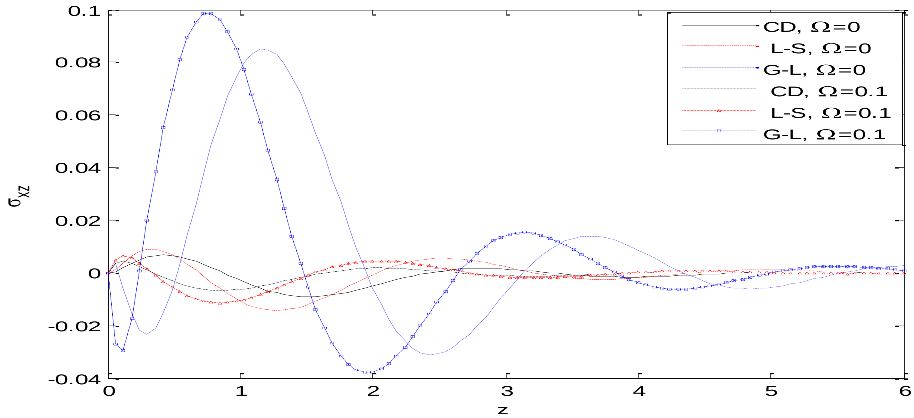

Fig. 14 Distribution of stress component σ_{xz} in the absence and presence of rotation.

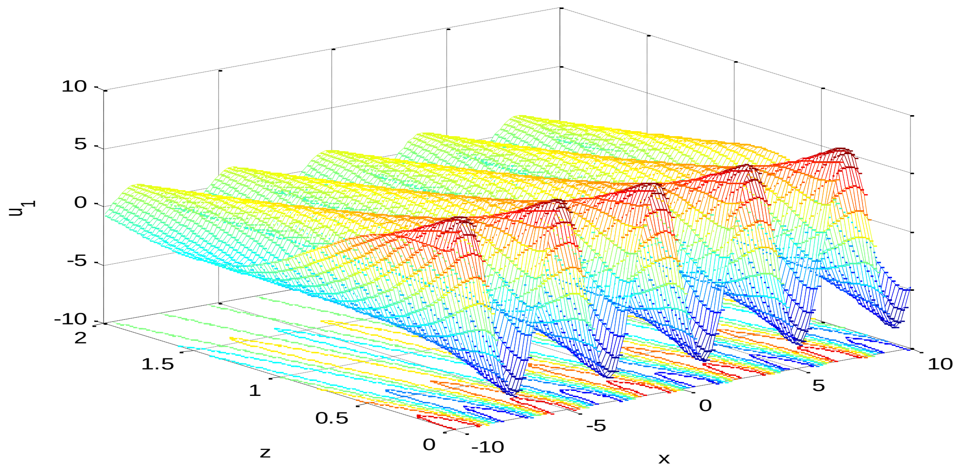

Fig. 15 (3D) Distribution of the displacement component u_1 against both components of distance based on (G-L) model at

$$\Omega = 0.1 rad / s,\ g = 9.8\ m / s^2.$$

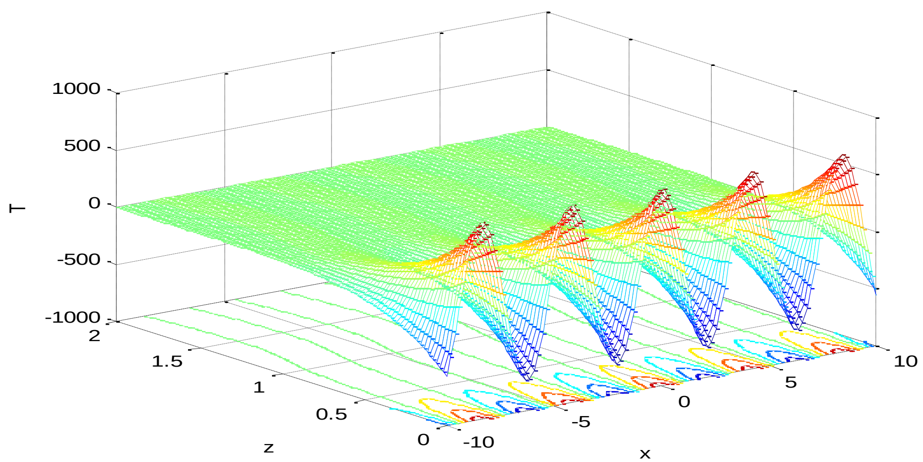

Fig. 16 (3D) Thermodynamic temperature distribution T against both components of distance based on (G-L) model at

$$\Omega = 0.1 rad/s,\ g = 9.8\ m/s^2.$$

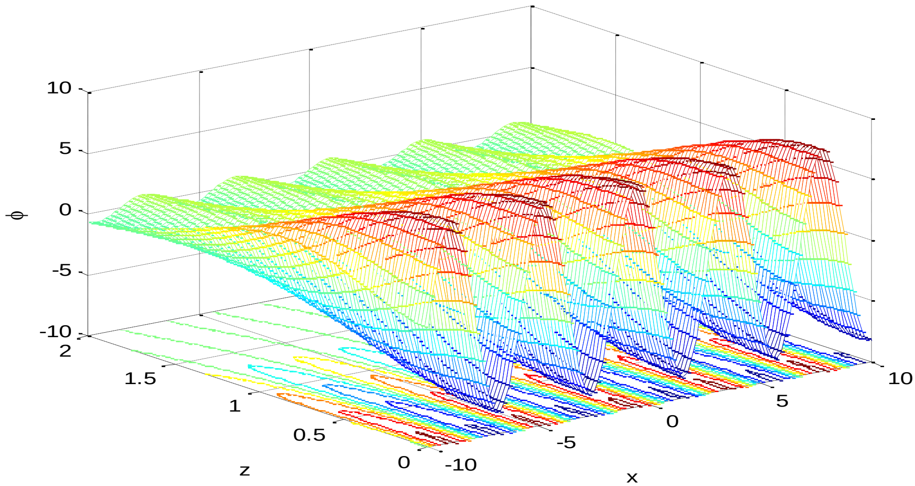

Fig. 17 (3D) Distribution of the change in volume fraction field ϕ against both components of distance based on (G-L) model at $\Omega = 0.1 rad/s,\ g = 9.8\ m/s^2$.

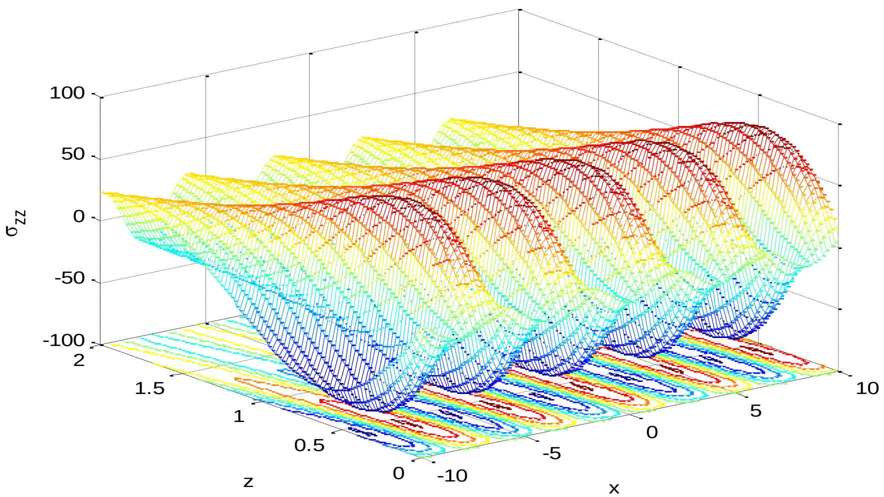

Fig. 18 (3D) Distribution of the stress component σ_{zz} against both components of distance based on (G-L) model at $\Omega = 0.1 rad/s,\ g = 9.8\ m/s^2$.

7. Conclusions

By comparing the figures that were obtained for the three thermoelastic theories, important phenomena are observed:

1. The values of all physical quantities converge to zero with increasing distance x, and all functions are continuous.
2. . The phenomenon of finite speeds of propagation is manifested in all figures.
3. All physical quantities satisfy the boundary conditions.
4. The thermoplastic materials with voids have an important role in the distribution of the field quantities.
5. the rotation and gravity have a great role in all considered physical quantities, since the amplitudes of these quantities is varying (increasing or decreasing) with the increase of the rotation and the gravity values.
6. Analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed and utilized.
7. Finally it deduced that the deformation of a body depends on the nature of the applied forces and gravity effect as well as the type of boundary conditions. z

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