إستمرارية مقياس الحد والرواسم شبه الامتثالية التوافقية على كرة الوحدة في الفضاء $\mathbb{R}^n$

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الملخص:

في هذه الورقة، نثبت لـ majorant ما ω الخاصية
$|f(x)-f(y)| \leq \omega(|x-y|)$; $x,y \in \partial\mathbb{B}$
حيث $f : \overline{\mathbb{B}} \longrightarrow \mathbb{R}^n$ تكون راسم شبه امتثالي توافقي مستمر في كرة الوحدة $\mathbb{B}$ تؤدي الى الخاصية المقابلة

$$|f(x)-f(y)| \leq C\,\omega(|x-y|),\ \ x,y \in \mathbb{B}.$$

هنا C يكون ثابت يعتمد فقط على كل من n و $K(f)$ و $\mathrm{diam}(\mathbb{B})$.

BOUNDARY MODULUS OF CONTINUITY AND HARMONIC QUASICONFORMAL MAPPINGS ON THE UNIT BALL IN $\mathbb{R}^n$

Abstract.

In this paper, we prove that for some majorant ω, the property
$|f(x)-f(y)| \leq \omega(|x-y|)$; $x,y \in \partial\mathbb{B}$
where $f : \overline{\mathbb{B}} \longrightarrow \mathbb{R}^n$ is a continuous mapping which is harmonic quasiconformal in $\mathbb{B}$ implies the corresponding property

$$|f(x)-f(y)| \leq C\,\omega(|x-y|),\ \ x,y \in \mathbb{B}.$$

Here C is a constant depends only on n; $K(f)$; and diam($\mathbb{B}$)
.

1. Introduction

Let $\mathbb{B} = \{x \in \mathbb{R}^n : |x| < 1\}$ be open unit ball in $\mathbb{R}^n$; $n > 2$, and $\partial\mathbb{B}$ be the boundary of $\mathbb{B}$. Harmonic quasiregular (briey, hqr) mappings in the plane were studied first by O.

Martio in [8], for a review of this subject and further results see [9] and references cited there. Moduli of continuity of harmonic quasiregular mappings in $\mathbb{B}^n$ were studied by several authors; see [7], [6], [3],Moduli of continuity of harmonic quasiregular mapping on bounded domain was studied by A. Abaoub, A. Shkheam, M. Arsenović, and M. Mateljević in[2]. We consider majorization results for function f that is quasiconformal in unit ball $\mathbb{B}$ of the Euclidean n-space $\mathbb{R}^n$, where $n \geq 2$. If $\omega: [0,+\infty) \longrightarrow [0,+\infty)$ nondecreasing function defined for $t \geq 0$ satisfies the condition

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$$\omega(A\,t) \leq A\,\omega(t) \quad \text{for all} \quad t \geq 0, \qquad (1)$$

for some fixed $A > 1$, we say that ω is majorant. More general,
a subadditive function ω satisfies (1) whenever A is a positive integer. Note that we may have $\omega(0) > 0$, and that ω need not be continuous. We remark that if (1) holds, then

$$\omega(ALt) \leq AL\omega(t) \quad \text{for all } t \geq 0; \text{ and } L \geq 1 \qquad (2)$$

In matters regarding notation and terminology we will conform to the usage in the book of Väisälä. In particular, $f : \mathbb{B} \longrightarrow \mathbb{R}^n$ is quasic- -onformal, $K_I = K_I(f)$ denotes the inner dilatation of f, $K_O = K_O(f)$ denotes the outer dilatation of f, and $K(f)$ designate the maximal dilatation of f.

2. Auxiliary Result.

The following result is contained in theorem 4 (see [1]).

Lemma 1.

Let $f : \mathbb{B} \longrightarrow \mathbb{R}^n$ is a continuous mapping which is quasiconformal in $\mathbb{B}$ and satisfies

$$|f(x) - f(y)| \leq \omega(|x - y|); \qquad (3)$$

for all $x; y \in \partial\,\mathbb{B}$, and for some majorant ω. Then

$$|f(x) - f(y)| \leq C\omega(|x - y|); \qquad (4)$$

for all $x \in \partial\,\mathbb{B}$, and all $y \in \mathbb{B}$, where C is a constant depending only on n; $K(f)$, and diam($\mathbb{B}$).

The following was proved in [5].

Lemma 2.

Let Ω be a bounded open subset of $\mathbb{R}^n$. Assume that f be a continuous mapping on $\overline{\Omega}$, and harmonic in Ω. If for each $x_0 \in \partial\,\Omega$

$$\sup_{\mathbb{B}(x_0,\rho)\cap\Omega} |f(x) - f(x_0)| \leq \gamma(\rho), \quad for\ \rho \leq \rho_0 \qquad (5).$$

Then for $x; y \in \Omega$,

$$|f(x) - f(y)| \leq \gamma(|x - y|); \text{whenever } |x - y| \leq \rho_0 \qquad (6).$$

3. Main Result

In this section, we will prove the main result in this paper.

Theorem 1.

Let $f : \mathbb{B} \longrightarrow \mathbb{R}^n$ be a continuous mapping which is harmonic quasiconformal in $\mathbb{B}$. If

$$|f(x) - f(y)| \leq \omega(|x - y|) \qquad (7)$$

for all $x; y \in \partial\,\mathbb{B}$, and for some majorant ω. Then

$$|f(x) - f(y)| \leq C\omega(|x - y|) \qquad (8)$$

for all $x, y \in \mathbb{B}$.

Proof.

By Lemma (1), estimate (8) holds for all $x \in \partial\mathbb{B}$ and all $y \in \mathbb{B}$. Using lemma (2), we deduced that the same estimate is valid for all $x; y \in \mathbb{B}$. ■

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