

# Calculating the Energy Distribution $f_e(E)$ of the Plasma Electrons Using the Maxwell Distribution for Some Isotopes

Abtisam A. Alakrout (\*)

*Dept. of environment and earth sciences, faculty of sciences*

## Abstract:

*In this paper, we studied the evaluation and statistical distribution of the energy of plasma electrons for a group of isotopes, which starts from Hafnium ( $^{176}_{72}\text{Hf}$ ) and ends at osmium ( $^{186}_{76}\text{Os}$ ). We have used Maxwell's distribution and Based on the results obtained from the experimental work, proved that the temperature of the plasma is greater than the capacity of ionization of the upper electronic level. In addition, by the program MATLAB was used for evaluating the energy distribution*

---

(\*) Email: ebtisamalkroot55@gmail.com

$f_e(E)$  of the plasma electrons. Illustrative examples are given to show the accuracy and reliability of the results.

**Keywords:** Energy distribution; Probability; Maxwell distribution ; Atomic levels; Electron energy .

### الملخص :

في هذا البحث قدمت دراسة تقييمية واحصائية لتوزيع الطاقة للإلكترونات البلازما باستخدام توزيع ماكسويل لبعض النظائر والتي تبدأ من عنصر الهافنيوم ( $^{176}_{72}\text{Hf}$ ) وحتى عنصر الأوزميوم ( $^{186}_{76}\text{Os}$ ) . حيث تم اثبات أن درجة حرارة البلازما أكبر من قدرة التأين الخاصة بالمستوي الإلكتروني العلوي. تم استخدام البرنامج (MATLAB) لتقييم توزيع الطاقة  $f_e(E)$  للإلكترونات البلازمية ، وقدمت أمثلة توضيحية تبين دقة النتائج .

**كلمات البحث:** توزيع الطاقة ، الاحتمالية ، توزيع ماكسويل ، المستويات الذرية ، طاقة

الإلكترون .

## 1. Introduction

The electron energy distribution function  $f_e(E)$  plays an important role in plasma modeling. Various approaches can be used to describe the  $f_e(E)$ , such as Maxwellian, Druyvesteyn, or using a solution of the Boltzmann equation[1]. The electron energy distribution function  $f_e(E)$  is essential in plasma modeling because it is needed to compute reaction rates for electron collision reactions. Because electron transport properties can also be derived from the  $f_e(E)$ , the choice of the  $f_e(E)$  you use influences the results of the plasma model. If the plasma is in thermodynamic equilibrium, the  $f_e(E)$  has a Maxwellian shape. In most

plasmas, for technical purposes, deviations from the Maxwellian form occur[1].

To describe the  $f_e(E)$ , several possibilities are available, such as a Maxwell or Druyvesteyn function. In addition, a generalized form is available, which is an intermediate between the Maxwell and the Druyvesteyn function. If the electrons are in thermodynamic equilibrium among each other, the distribution function is Maxwellian. However, this is only true if the ionization degree is high. Here, electron-electron collisions drive the distribution towards a Maxwellian shape. Inelastic collisions of electrons with heavy particles lead to a drop of the  $f_e(E)$ , at higher electron energies. [1]

Normally, the distribution function is divided by for illustration purposes. This kind of distribution function is also known as an electron energy probability function ( $f_e(E)$ ). For

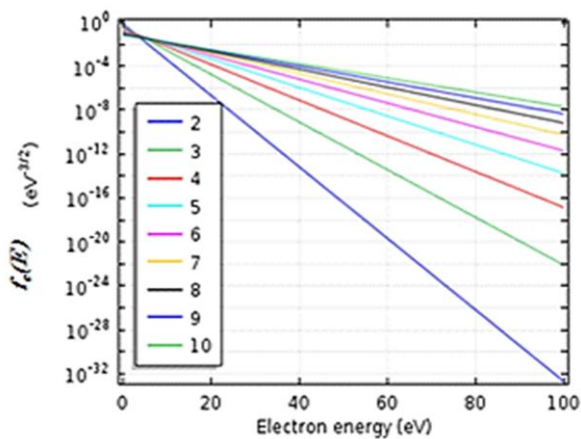
The rate coefficients for excitation and ionization highly depend on the shape of the  $f_e(E)$ . This is due to the exponential drop-off in the population of electrons at energies exceeding the activation threshold. Using a Maxwellian  $f_e(E)$  can lead to an overestimation of the ionization rate, which is shown below.

## 2. The Maxwell–Boltzmann distribution

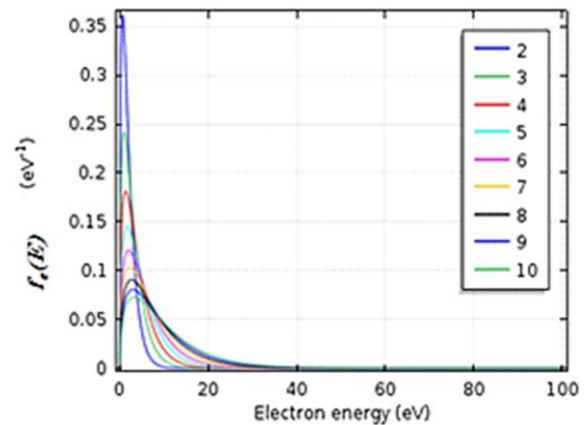
The distribution was first derived by Maxwell in 1860 on heuristic grounds.[4] Boltzmann later, in the 1870s, carried out significant investigations into the physical origins of this distribution. Normally, the distribution function is divided by for illustration purposes. This kind of distribution function is also known as an electron energy probability function ( $f_e(E)$ ). For a Maxwellian function, this results in a straight line with a slope of ,as in Fig(1)(a,b)[1] .The Maxwell–Boltzmann distribution is a particular probability distribution named after James Clerk Maxwell

and Ludwig Boltzmann. It was first defined and used for describing particle speeds in idealized gases, where the particles move freely inside a stationary container without interacting with one another, except for very brief collisions in which they exchange energy and momentum with each other or with their thermal environment. The term "particle" in this context refers to gaseous particles only (atoms or molecules), and the system of particles is assumed to have reached thermodynamic equilibrium.[2] The energies of such particles follow what is known as Maxwell-Boltzmann statistics, and the statistical distribution of speeds is derived by equating particle energies with kinetic energy.

Mathematically, the Maxwell–Boltzmann distribution is the chi distribution with three degrees of freedom (the components of the velocity vector in Euclidean space), with a scale parameter measuring speeds in units proportional to the square root of  $(T/m)$  (the ratio of temperature and particle mass).[3]



Fig(1)a: Maxwellian  $f_e(E)$  in  $eV^{-3/2}$  for mean electron energies from 2 — 10 eV



Fig(1)b: Maxwellian  $f_e(E)$  in  $eV^{-1}$  for mean electron energies from 2 — 10 eV

The Maxwell–Boltzmann statistics gives the average number of particles found in a given single-particle microstate. Under certain assumptions, the logarithm of the fraction of particles in a given microstate is proportional to the ratio of the energy of that state to the temperature of the system:

$$-\log\left(\frac{N_i}{N}\right) \propto \frac{E_i}{T} \quad (1)$$

The assumptions of this equation are that the particles do not interact, and that they are classical; this means that each particle's state can be considered independently from the other particles' states. Additionally, the particles are assumed to be in thermal equilibrium.[2,5] The  $f_e(E)$  can be computed by solving the Boltzmann equation. The Boltzmann equation describes the evolution of the distribution function, , in a six-dimensional phase space.

This relation can be written as an equation by introducing a normalizing factor:

$$f_e(E) = \sqrt{\frac{4E_e}{\pi T}} \exp\left(\frac{-E_e}{T}\right), \quad (2)$$

[  $T=E_N$  ] and  $E_e$  is the electron energy, (keV)

To solve the Maxwell–Boltzmann equation and, therefore, compute the  $f_e(E)$  , drastic simplifications are necessary. A common approach is to expand the distribution function in spherical harmonics. The  $f_e(E)$  is assumed to be almost spherically symmetric, so the series can be truncated after the second term (a so-called two-term approximation). This approach is the most accurate way to compute the  $f_e(E)$  because an

anisotropic perturbation, due to inelastic collisions, is taken into account. However, this is also the most computationally expensive approach.

It is demonstrated that if the plasma temperature (T) is greater than the ionization potential of the upper electronic level, then the probability of finding an electron and a hole at the upper and lower atomic levels respectively, S is given by [6].

By the equipartition theorem, this energy is evenly distributed among all three degrees of freedom, so that the energy per degree of freedom is distributed as a chi-squared distribution with one degree of freedom:[7,8]

$$S \approx n_e (\lambda_T / 2\pi)^3, \quad (3)$$

Where  $n_e$  is the electron density in the plasma

$$(n_e \approx 10^{20} \text{ cm}^{-3})$$

and

$$\left( \lambda_T = 2\pi / \sqrt{2m_e T} \right) \quad (4)$$

Is the thermal wavelength of the plasma electron and  $m_e$  is the electron mass .

#### 4- Results & Discussion

Data was processed using the program MATAB. Where statistical equations were obtained and different forms showing the relationship between the distribution of the energy of plasma electrons  $f_e(E)$  and the electron energy ( $E_N$ ) and the transition energy ( $E_e$ ). As we mentioned in the theory that the entire theoretical procedures and equations are given in our research. Where we present how to calculate the energy distribution of the plasma electrons. Where as the mean equation used in calculating

the energy distribution is equ.(2), to be notice . Since our theoretical model is simple , it is not necessary for us to know the details . It is sufficient for us to consider only the energy balance from the list of orbital binding energy in tables .

The Maxwell distribution  $f_e(E)$  varies directly with the square root of the electron energy and inversely with the square root of the plasma temperature, so the distribution, reaches the peak at the higher electron energies and at lower plasma temperature (  $T \sim E_N$  ). From **Table .(1)**, the Maxwell distribution varies in the range from 4.0E-07for  $^{180}_{72}Hf$  to 0.869 for  $^{181}_{73}Ta$ .

The probability S of finding an electron and a hole at the upper and lower atomic levels, respectively , varies directly with the thermal wavelength of the plasma electron  $\lambda_T$  to the power (3) in turn proportional inversely with the plasma temperature (  $T \sim E_N$  ), this probability decreases with the transition energy. From the same table, the probability of finding an electron and a hole at the upper and lower atomic levels respectively (S) varies in the range from 70.11E-32 for  $^{182}_{74}W$  with a transition energy of 222.108KeV and a multipolarity of E1 to 30.72E-07 for  $^{181}_{73}Ta$  with a transition energy of 6.238 KeV and transition multipolarity of E1

From the graphical representation of **fig(2)a**, the Maxwell distribution  $f_e(E)$  varies with the electron energy and the nuclear transition for different atomic numbers so the distribution reaches a peak at the higher electron energy and a lower plasma temperature (  $T \sim E_N$  ).

From the graphical representation of **fig(2)b**, the probability ( S ) increases with the electron energy and decreases with the transition energy for different atomic numbers.

**Table (1). The Maxwell distribution of the plasma electrons  $f_e(E)$  and the probability S at different plasma temperature**

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron $\sim E_e$ (keV)	$f_e(E)$	S
$^{176}_{72}\text{Hf}$	E2	88.34 3	E=4 E=8 E=12	0.162 0.309 0.363	11.13E-10
$^{177}_{72}\text{Hf}$	E2	112.950	E=4 E=8 E=12	0.204 0.279 0.332	53.29E-11
$^{177}_{72}\text{Hf}$	E2	128.503	E=4 E=8 E=12	0.193 0.265 0.314	36.17 E-11
$^{177}_{72}\text{Hf}$	E2	153.284	E=4 E=8 E=12	0.177 0.245 0.292	21.35 E-11
$^{177}_{72}\text{Hf}$	E2	204.105	E=4 E=8 E=12	0.155 0.214 0.258	9.06 E-11
$^{177}_{72}\text{Hf}$	E2	208.366	E=4 E=8 E=12	0.156 0.213 0.256	8.52 E-11
$^{177}_{72}\text{Hf}$	E4	214.434	E=4 E=8 E=12	0.151 0.210 0.252	7.75 E-11
$^{180}_{72}\text{Hf}$	E1	57.555	E=4 E=8 E=12	0.081 0.366 0.418	40.32 E-10
$^{180}_{72}\text{Hf}$	E2	93.326	E=4 E=8 E=12	0.224 0.303 0.357	9.45 E-10



Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron $\sim E_e$ (keV)	$f_e(E)$	S
$^{180}_{72}\text{Hf}$	E2	215.256	E=4 E=8 E=12	0.151 0.257 0.252	7.68 E-11
$^{180}_{72}\text{Hf}$	E2	332.277	E=4 E=8 E=12	0.870 0.541 0.654	20.74 E-12
$^{180}_{72}\text{Hf}$	E2	443.09	E=4 E=8 E=12	4.0 E-07 0.471 0.970	8.83 E-12
$^{180}_{72}\text{Hf}$	E3	500.64	E=4 E=8 E=12	1.08 E-06 0.444 0.539	61.21 E-13

Table.(1). Continued

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron $\sim E_e$ (keV)	$f_e(E)$	S
$^{181}_{73}\text{Ta}$	E1	6.238	E=4 E=8 E=12	0.476 0.355 0.229	30.72 E-07
$^{181}_{73}\text{Ta}$	E3	133.024	E=4 E=8 E=12	0.189 0.2608 0.3098	32.64 E-11
$^{181}_{73}\text{Ta}$	E4	615.21	E=4 E=8 E=12	0.869 0.020 0.890	32.95 E-13

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron $\sim E_e$ (keV)	$f_e(E)$	S
$^{182}_{74}\text{W}$	E1	31.738	E=4 E=8 E=12	0.353 0.441 0.476	24.10 E-09
$^{182}_{74}\text{W}$	E2	65.711	E=4 E=8 E=12	0.259 0.349 0.401	26.88 E-10
$^{182}_{74}\text{W}$	E1	67.750	E=4 E=8 E=12	0.258 0.345 0.398	24.73 E-10
$^{182}_{74}\text{W}$	E2	84.680	E=4 E=8 E=12	0.233 0.316 0.369	13.10 E-10
$^{182}_{74}\text{W}$	E2	100.106	E=4 E=8 E=12	0.217 0.295 0.347	38.17 E-11
$^{182}_{74}\text{W}$	E2	113.672	E=4 E=8 E=12	0.204 0.279 0.329	52.22 E-11
$^{182}_{74}\text{W}$	E1	116.418	E=4 E=8 E=12	0.202 0.276 0.327	48.38 E-11
$^{182}_{74}\text{W}$	E1	152.430	E=4 E=8 E=12	0.178 0.246 0.293	21.50 E-11
$^{182}_{74}\text{W}$	E1	156.386	E=4 E=8 E=12	0.176 0.242 0.289	19.96 E-11

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron- $E_e$ (keV)	$f_e(E)$	$S$
$^{182}_{74}\text{W}$	E1	222.108	E=4 E=8 E=12	0.148 0.206 0.248	70.11 E-32
$^{183}_{74}\text{W}$	E2	46.484	E=4 E=8 E=12	0.304 0.394 0.443	76.49 E-10

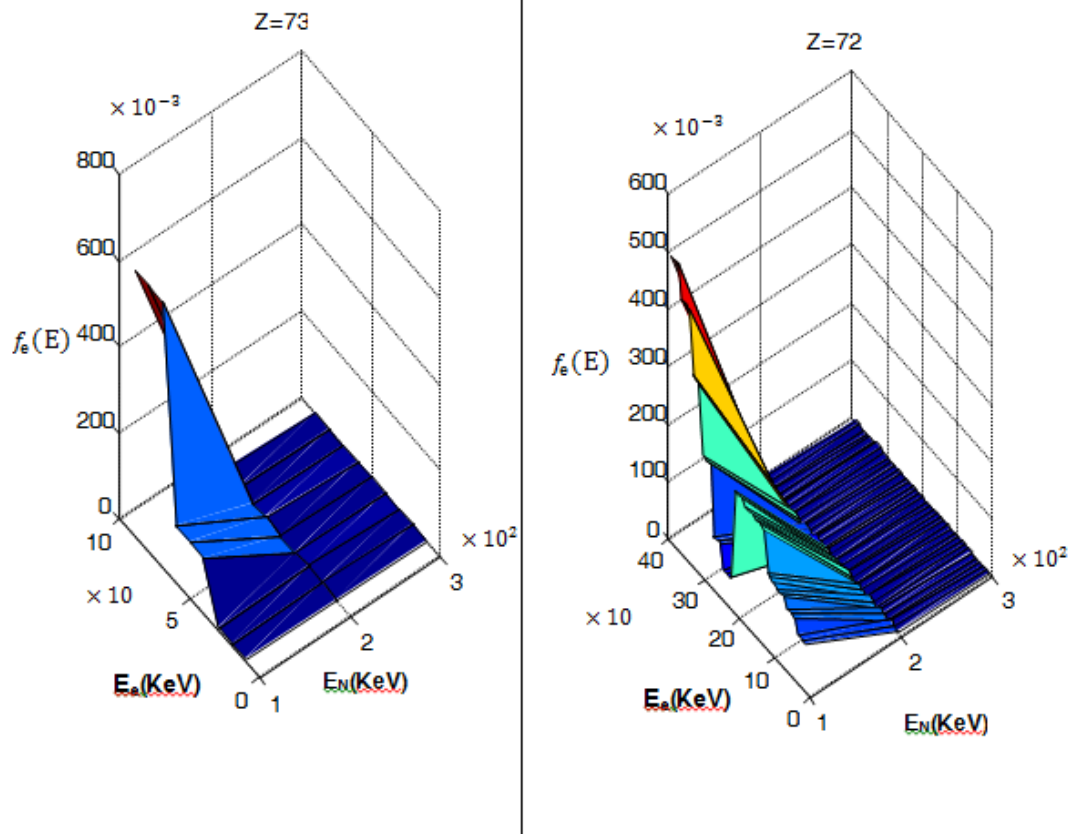
Table.(1). Continued

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron- $E_e$ (keV)	$f_e(E)$	$S$
$^{182}_{74}\text{W}$	E2	52.595	E=4 E=8 E=12	0.288 0.378 0.519	52.99 E-10
$^{183}_{74}\text{W}$	E2	99.079	E=4 E=8 E=12	0.217 0.296 0.348	7.91 E-10
$^{183}_{74}\text{W}$	E2	109.726	E=4 E=8 E=12	0.208 0.284 0.334	58.37 E-11
$^{183}_{74}\text{W}$	E2	192.645	E=4 E=8 E=12	0.016 0.220 0.265	10.75 E-11
$^{183}_{74}\text{W}$	E2	291.724	E=4 E=8 E=12	0.130 0.182 0.219	30.72 E-12

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron $\sim E_e$ (keV)	$f_e(E)$	$S$
$^{185}_{75}\text{Re}$	M1	234.157	E=4 E=8 E=12	0.145 0.202 0.242	59.90 E-12
$^{187}_{75}\text{Re}$	E1	72.001	E=4 E=8 E=12	0.218 0.336 0.391	20.70 E-10
$^{187}_{75}\text{Re}$	E2	106.596	E=4 E=8 E=12	0.211 0.287 0.363	63.70 E-11
$^{187}_{75}\text{Re}$	M1	113.747	E=4 E=8 E=12	0.205 0.279 0.330	52.22 E-11
$^{187}_{75}\text{Re}$	E2	134.243	E=4 E=8 E=12	0.190 0.260 0.308	41.49 E-11

Table.(1). Continued

Nuclide	Transition multipolarity[8]	$T \approx E_N(\text{keV})$ [8]	Energy of electron $\sim E_e$ (keV)	$f_e(E)$	$S$
$^{188}_{75}\text{Re}$	E2	208.844	E=4 E=8 E=12	0.153 0.211 0.256	8.43 E-11
$^{186}_{76}\text{Os}$	E2	137.157	E=4 E=8 E=12	0.187 0.258 0.306	29.95 E-11
$^{188}_{76}\text{Os}$	E2	155.032	E=4 E=8 E=12	0.176 0.243 0.291	20.70 E-11
$^{186}_{76}\text{Os}$	E4	74.379	E=4 E=8 E=12	0.248 0.338 0.386	18.43 E-11



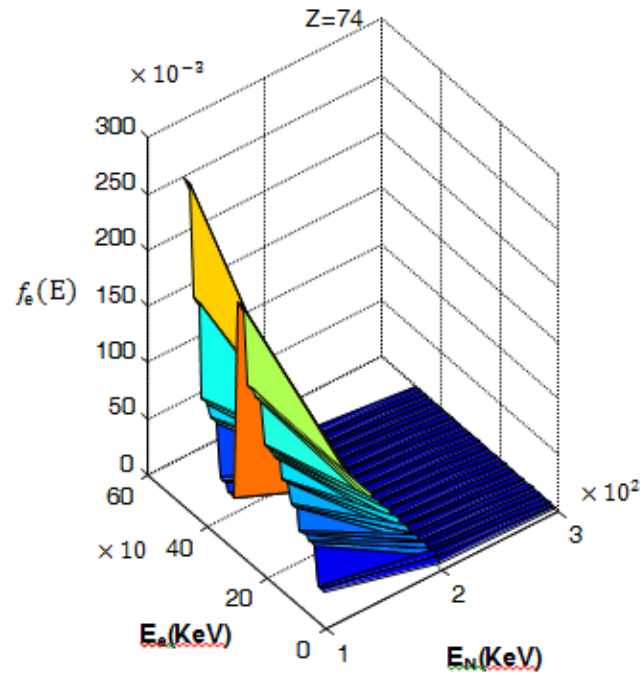
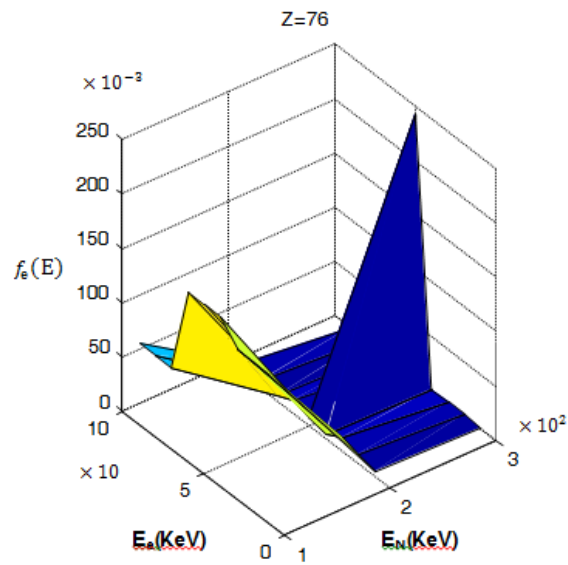
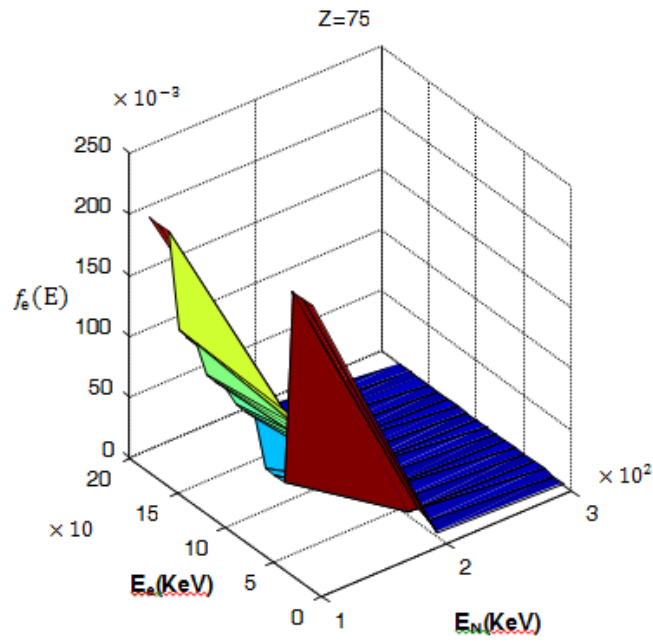
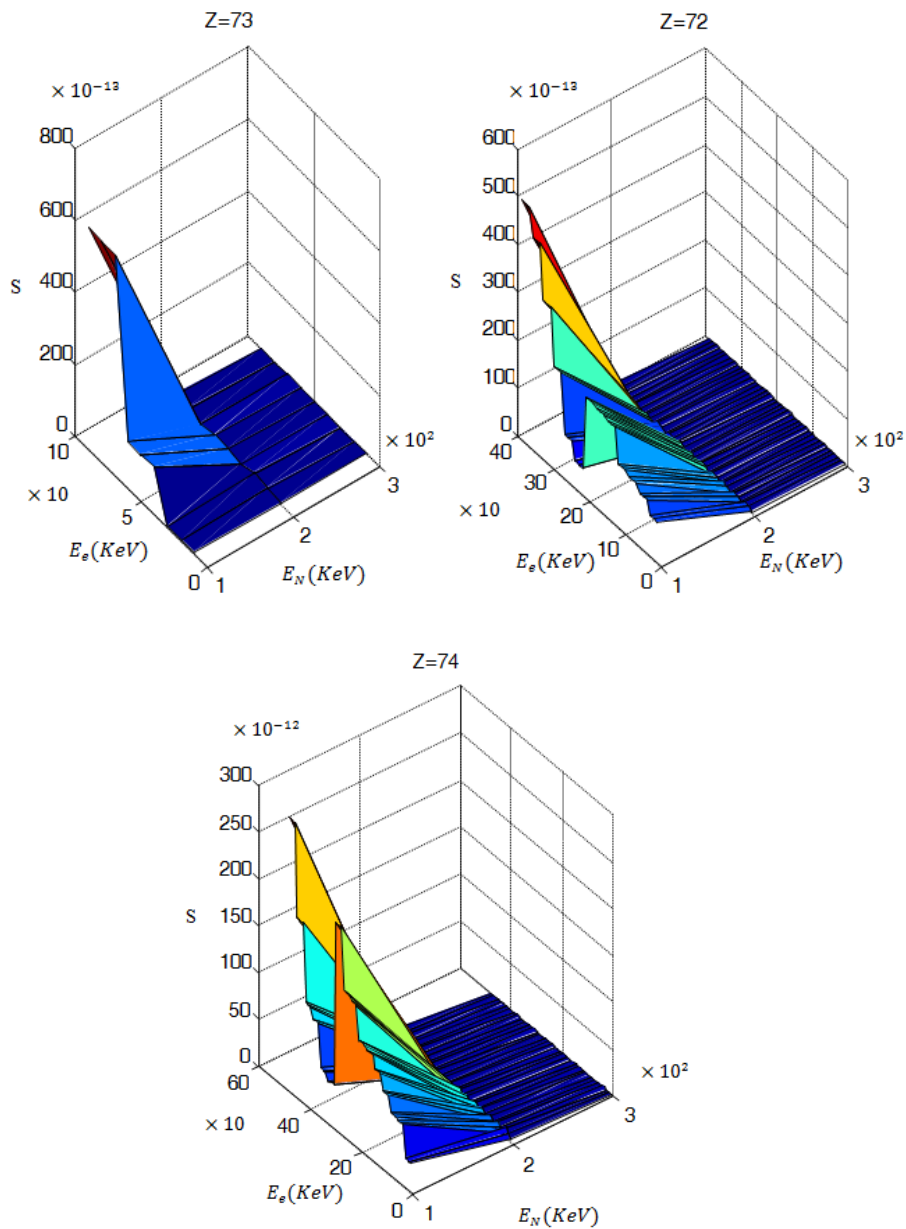


Fig.(2)a. The energy distribution  $f_e(E)$  of the plasma electron as a function of the normalized values of  $E_N$  and  $E_e$  for different  $Z$  number.

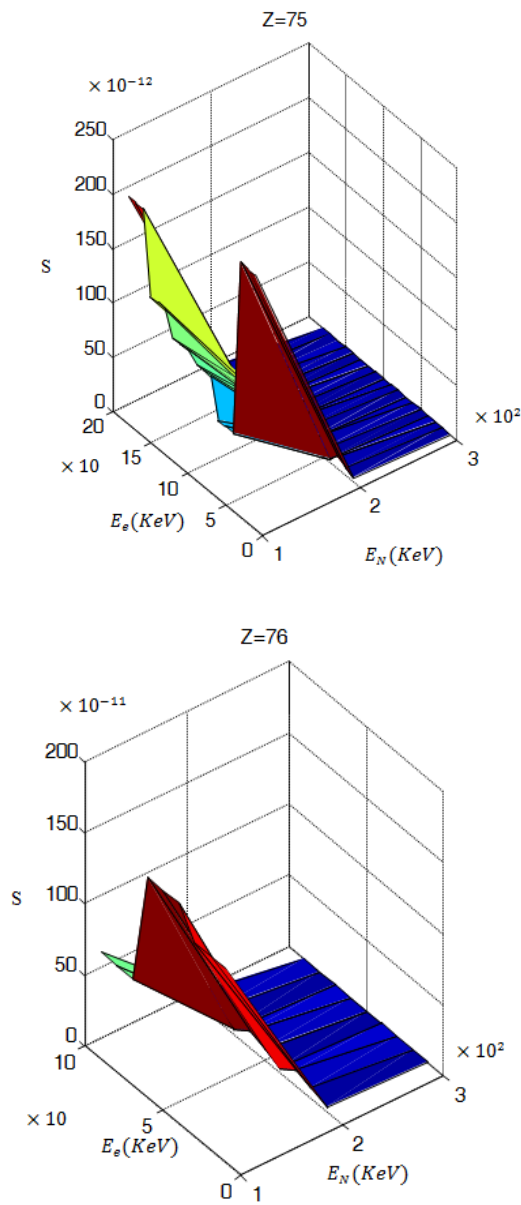


Fig(2)a.Continued.





**Fig(2)b .Variation of the probability S with the normalized values of  $E_N$  and  $E_e$  for different Z number.**



Fig(2)b.Continued.

## 5- Conclusions

In this paper, the agreement between theoretical and experimental levels is satisfactory for excitation energies and transition probabilities and the energy distribution  $f_e(E)$ . Where the Maxwell distribution varies directly with the square root of the electron energy and inversely with the square root of the plasma temperature. The Maxwell distribution varies in the range from 4.0E-07 for ( $^{180}_{72}\text{Hf}$ ) to 0.869 for ( $^{181}_{73}\text{Ta}$ ). The theoretical calculations for the energy distribution  $f_e(E)$  using the MATLAB for the evaluation and the graphical representation of the results reasonably agree with the experimental.

## References

- [1] Annette Pahl, *Phys. Rev. (Electron Energy Distribution Function)*, August 4, (2014)
- [2] *Statistical physics (2<sup>nd</sup> Edition)*, F. Mandl, Manchester physics, John wily & Sons, 2008.
- [3] *University physics-with Modern physics (12<sup>th</sup> edition)*, H.D Young, R.A.Freedman (Original edition), Addison-wesley (pearson international), 1<sup>st</sup> Edition: 1949, 12<sup>th</sup> Edition 2008.
- [4] Maxwell, J.C. (1860 A): *Illustrations of the dynamical theory of gases. Part I. On the motions and collisions of perfectly elastic spheres. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 4th Series, vol.19, pp.19-32*
- [5] *McGraw Hill Encyclopaedia of Physics (2nd Edition)*, C.B. Parker, 1994, ISBN 0-07-051400-3
- [6] G. Pretzler, A. Saemann, A. Pukhov et al., *Phys. Rev. E* 58, 1165 (1998).

[7] Laurendeau, Normand M. (2005). *Statistical thermodynamics: fundamentals and applications*. Cambridge University Press. p. 434. ISBN 0-521-84635-8., Appendix N, page 434

[8] S. Raman et al., *Atomic and Nuclear Data Tables* 92(2006)207-243.