دراسة تأثير كل من الدوران والمجال المغناطيسي في وجود درجتي حرارة باستخدام نظريات مختلفة للمرونة الحرارية

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الملخص العربي :

في هذا البحث تم استعراض معادلات المرونة الحرارية المعممة لجسم مرن حراريًا متماثل موحد الخواص تحت تأثير كل من الدوران والمجال المغناطيسي في وجود درجتي حرارة: إحداهما تسمي درجة الحرارة الديناميكية، والأخرى تسمي درجة الحرارة الموصلة للوسط وذلك في ضوء خمس نظريات للمرونة الحرارية وهي: النظرية المرتبطة (CT), نظرية لورد وشولمان (L-S) ذات الزمن الإسترخائى الواحد, نظرية جرين (G-L) وليندساي ذات زمني الإسترخاء, نظرية جرين وناخدى من النوع الثاني (G-N II) ونموذج تسو (DPL). تم تطبيق طريقة تحليل السلوك العادي لإيجاد الكميات الفيزيائية المختلفة وهي طريقة تستخدم للحصول علي الحلول المضبوطة للمسائل الرياضية تحت الدراسة وهي تعتمد علي فصل المتغيرات، حيث تقوم بتحويل المعادلات التفاضلية الجزئية إلي معادلات تفاضلية عادية وتكون بذلك قدمت نتائج مضبوطة لحلول المسائل بدون أية فرضيات رياضية إضافية، بالإضافة إلي سهولة تعامل الحاسب الآلي مع مخرجات هذه الطريقة والرسم بطريقة واضحة وسهلة, وتم رسم هذه الكميات ومقارنتها في وجود وعدم وجود كل من الدوران والمجال المغناطيسي وكذلك في وجود وعدم وجود البارميتر الخاص بدرجتي الحرارة في ضوء نظريات (G-L, G-N II, DPL).

والنتائج التي خرج بها هذا البحث توضح مدى الفرق بين النظريات محل الدراسة وبواسطة عمل مقارنات بيانية للنتائج في ضوء تلك النظريات حيث يمكن القول بأن كل القيم من المعادلات الفيزيائية تتقارب إلى الصفر ومحققة لشروط الحدية وأيضا كل الدوال تكون مستمرة و كما يوضح هذا العمل إن الدوران والمجال المغنطيسي لهما دور مهم في توزيع كميات الحقل باستثناء الحرارة وأيضا يوضح تأثير سرعة المصدر الحراري الموجود في الوسط على سرعة تقدم الموجات الحرارية والموجات الميكانيكية.

ونشير إلـي أن جميـع النتـائج التـي تـم الحصـول عليهـا والرسـم كانـت باسـتخدام برنامج (Matlab R2013a).

Influence of rotation and generalized magneto-thermoelastic medium with two temperature under different theories

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Abstract: This paper studies the two dimensional problem of thermoelastic rotating material under the effect of magnetic field and a two temperature generalized thermoelasticity in the context of five theories of generalized thermoelasticity: Lord-Schulman with one relaxation time, Green-Lindsay with two relaxation times, Green-Naghdi theory (of type II) without energy dissipation and Chandrasekaraiah-Tzou theory with dual phase lags, as well as the coupled theory. The normal mode analysis is used to obtain the exact expressions for the considered variables. Some particular cases are also discussed in the context of the problem. Numerical results for the considered variables are obtained and illustrated graphically. Comparisons are also made with the results predicted by different theories (G-L, G-N II, DPL) in the absence and presence of rotation, magnetic field, as well as two temperature parameters.

Keywords- generalized thermo-elasticity; Magnetic field; Rotation; Conductive temperature; Normal mode analysis.

1. INTRODUCTION

The generalized thermoelasticity theories have been developed with the aim of removing the paradox of infinite speed of heat propagation inherent in the classical
coupled dynamical thermoelasticity theory investigated by Biot [6].The first attempt
towards the introduction of generalized thermoelasticity was started by Lord and Shulman [21], who formulated the theory by incorporating a flux-rate term into conventional Fourier's law of heat conduction. The L-S (Lord-Shulman) theory introduces a new physical concept which called a relaxation time. Since the heat conduction equation of this theory is of the wave-type, it automatically ensures finite speed of propagation for heat wave. The second generalization was developed by Green and Lindsay [16]. This theory contains two constants that act as relaxation
times and modifies all the equations of coupled theory, not the heat conduction equation only. It is based on a form of the entropy inequality proposed by Green and Laws [15]. It does not violate the Fourier's law of heat conduction when the body under consideration has a center of symmetry, and it is valid for both isotropic and anisotropic bodies. The theory of thermoelasticity without energy dissipation is another generalized theory and was formulated by Green and Naghdi[17]. It includes the thermal-displacement gradient among its independent constitutive variables, and differs from the previous theories in that it does not accommodate dissipation of thermal energy.

Chandrasekharaiah [9] and Tzou [27] proposed dual-phase-lag thermoelasticity in 1998. A survey of five different thermoelastic models in which disturbances are transmitted in a wavelike manner is due to Hetnarski and Ignaczak [18].

Some researches in the past have investigated different problems of rotating media. In a paper by Schoenberg and Censor [26], the propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. Many author [8], [14] studied the effect of rotation on elastic waves. These problems are based on the more realistic elastic model since earth, the moon and other planets have angular velocity.

The theory of magneto-thermoelasticity is concerned with the interacting effects of applied magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial appliances, particularly in nuclear devices, where there exists a primary magnetic field; various investigations are to be carried out by considering the interaction between magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies. The development of the interaction of electromagnetic field, the thermal field and the elastic field is available in many works such as Othman [22]. Problems related to magneto-thermoelasticity with thermal relaxation times have been investigated by Othman and Singh[21], Othman and Abbas[23], Abbas and A.M. Zenkour [4], [3], Abbas[2] and Abbas and Abo-Dahab[1].

Chen and Gurtin, [10] Chen et al. [11], [12] have formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature θ and the thermodynamic temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two temperatures are identical[10]. For time-dependent problems, however, and for wave propagation problems in particular, the two temperatures are in general different, regardless of the presence of a heat supply. The two temperatures T, θ and the strain are found to have representations in the form of a traveling wave plus a response, which occurs instantaneously throughout the body [7], and Warren and Chen [28] investigated the wave propagation in the two-temperature theory of thermoelasticity. Recently, Youssef [20] investigated a two-temperature generalized thermoelasticity theory together with a general uniqueness theorem and solved many applications in the context of this theory[31], [32].

The paper deals with a specific organization form of matter. Other forms and description are given for example in the Refs. [19],[25],[33]. In most cases quantum theory is necessary for the description of the organization forms of matter. But even the interpretation of modern quantum theory seems still to be an open question, as is demonstrated in Ref. [5], [13],[20] [29].

The present paper is to investigate the effect of the rotation and the magnetic field on the plane waves in a linearly generalized thermoelastic isotropic medium with a two temperature in the context of five theories. The normal mode analysis is used to obtain the exact expressions for the considered variables. The distributions of the considered variables are presented graphically. Numerical results for the field quantities are given and illustrated graphically in the presence and absence of the rotation, magnetic field and two-temperature parameter.

2. Formulation of the problem

Consider the problem of a rotating thermoelastic half-space $(x \geq 0)$. The elastic medium is permeated into a uniform magnetic field with a constant intensity $\mathbf{H} = (0, 0, H_0)$, acting parallel to the boundary plane (taken as the direction of the z - axis). The surface of a half-space is subjected to a thermal shock which is a function of y and t. Thus, all quantities considered are independent of z and the third component of the displacement vector vanishes.

When the body forces are neglected, the governing equations are:

(1) Strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad i, j = 1, 2. \tag{1}$$

Where the components of the displacement vector are $u_i = (u, v, 0)$.

(2) Stress-displacement relation

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma(1 + \upsilon_0 \frac{\partial}{\partial t})T]\delta_{ij}, \tag{2}$$

Heat conduction equation [93]:

$$K(n^* + \tau_1 \frac{\partial}{\partial t})\theta_{,ii} = \rho C_E (n_1 + \tau_0 \frac{\partial}{\partial t})\dot{T} + \gamma T_0 (n_1 + n_0 \tau_0 \frac{\partial}{\partial t})\dot{e}, \tag{3}$$

Such that,

$$T = \theta - a^* \theta_{,ii}, \tag{4}$$

Equation of motion

Since the medium is rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega \mathbf{n}$ where $\mathbf{n}$ is a unit vector representing the direction of the axis of the rotation, the equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [11]): centripetal acceleration $\mathbf{\Omega}\wedge(\mathbf{\Omega}\wedge\mathbf{u})$ due to time varying motion only and Corioli's acceleration $2\mathbf{\Omega}\wedge\dot{\mathbf{u}}$, then the equation of motion in a rotating frame of reference is

$$\rho[\ddot{u}_i + \{\mathbf{\Omega}\wedge\mathbf{\Omega}\wedge\mathbf{u}\}_i + 2(\mathbf{\Omega}\wedge\dot{\mathbf{u}})_i] = \sigma_{ji,j} + F_i, \quad i, j = 1,2,3. \tag{5}$$

Where F_i is the Lorentz force and is given by:

$$F_i = \mu_0(\mathbf{J}\times\mathbf{H})_{\mathrm{i}}. \tag{6}$$

The variation of the magnetic and electric fields are perfectly conducting slowly moving medium and are given by Maxwell's equations:

$$\operatorname{curl}\mathbf{h} = \mathbf{J} + \varepsilon_0\,\dot{\mathbf{E}}, \tag{7}$$

$$\operatorname{curl}\mathbf{E} = -\mu_0\,\dot{\mathbf{h}}, \tag{8}$$

$$\mathbf{E} = -\mu_0\,(\dot{\mathbf{u}}\times\mathbf{H}), \tag{9}$$

$$\operatorname{div}\mathbf{h} = 0. \tag{10}$$

Where σ_{ij} are the stress components, λ, μ are elastic constants, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the thermal expansion coefficient, δ_{ij} is Kronecker's delta, T is the temperature above the reference temperature T_0, K is the thermal conductivity, n^*, n_0, n_1 are parameters, $\tau_1, \tau_0, \upsilon_0$ are relaxation times, ρ is the density, C_E is the specific heat at constant strain, θ is the conductive temperature, u_i are the components of the displacement vector, e_{ij} are the components of the strain tensor, μ_0 is the magnetic permeability, ε_0 is the electric permeability, $\boldsymbol{J}$ is the current density vector, $\boldsymbol{E}$ is the induced electric field vector, $\boldsymbol{h}$ is the induced magnetic field vector and H_0 is the constant magnetic field.

Expressing components of the vector $\mathbf{J}=(J_1,J_2,J_3)$ in terms of the displacement by eliminating the quantities $\mathbf{h}$ and $\mathbf{E}$ from equation (7), thus yields

$$J_1=h_{,y}+\varepsilon_0\mu_0H_0\ddot{v},\quad J_2=-h_{,x}-\varepsilon_0\mu_0H_0\ddot{u},\quad J_3=0. \tag{11}$$

Substituting from Eq. (11) in Eq. (6), one can get

$$F_1=-\mu_0H_0h_{,x}-\varepsilon_0\mu_0^2H_0^2\ddot{u},\quad F_2=-\mu_0H_0h_{,y}-\varepsilon_0\mu_0^2H_0^2\ddot{v},\quad F_3=0. \tag{12}$$

From Eqs. (2) and (12) into Eq. (5), the equations of motion can be written as

$$\rho(\ddot{u}-\Omega^2u-2\Omega\dot{v})=\mu\nabla^2u+(\lambda+\mu)e_{,x}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T_{,x}-\mu_0H_0h_{,x}-\varepsilon_0\mu_0^2H_0^2\ddot{u}, \tag{13}$$

$$\rho(\ddot{v}-\Omega^2v+2\Omega\dot{u})=\mu\nabla^2v+(\lambda+\mu)e_{,y}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T_{,y}-\mu_0H_0h_{,y}-\varepsilon_0\mu_0^2H_0^2\ddot{v}. \tag{14}$$

From Eqs. (7)- (10), it can be concluded that:

$$h=-H_0e\,. \tag{15}$$

The constitutive relations, using Eq. (2), can be written as

$$\sigma_{xx}=(\lambda+2\mu)u_{,x}+\lambda v_{,y}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T\,, \tag{16}$$

$$\sigma_{yy}=\lambda u_{,x}+(\lambda+2\mu)v_{,y}-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T\,, \tag{17}$$

$$\sigma_{zz}=\lambda e-\gamma(1+\upsilon_0\frac{\partial}{\partial t})T\,, \tag{18}$$

$$\sigma_{xy}=\mu(u_{,y}+v_{,x}),\quad \sigma_{xz}=\sigma_{yz}=0. \tag{19}$$

For simplification, the following non-dimensional variables are used:

$$\{t',t_1',\tau_0',\upsilon_0'\}=\omega^*\{t,t_1,\tau_0,\upsilon_0\},\qquad u_i'=\frac{\rho c_0\omega^*}{\gamma T_0}u_i,\ x_i'=\frac{\omega^*}{c_0}x_i,$$

$$\{T',\theta'\}=\frac{\{T,\theta\}}{T_0},\qquad \sigma_{ij}'=\frac{\sigma_{ij}}{\gamma T_0},\qquad h'=\frac{h}{H_0},\qquad \Omega'=\frac{\Omega}{\omega^*},\qquad \text{i, j}=1,2,3 \tag{20}$$

Where, $\omega^*=\rho C_E c_0^2/K$ and $c_0^2=(\lambda+2\mu)/\rho.$

In terms of the non-dimensional quantities defined in (20) and using (15), the above governing equations take the form (dropping the primes over the non-dimensional variables for convenience)

$$\alpha\ddot{u}-\Omega^2u-2\Omega\dot{v}=(1-\beta)\nabla^2u+h_1\frac{\partial e}{\partial x}-(1+\upsilon_0\frac{\partial}{\partial t})\frac{\partial T}{\partial x}, \tag{21}$$

$$\alpha\ddot{v}-\Omega^2v+2\Omega\dot{u}=(1-\beta)\nabla^2v+h_1\frac{\partial e}{\partial y}-(1+\upsilon_0\frac{\partial}{\partial t})\frac{\partial T}{\partial y}, \tag{22}$$

$$(n^*+\tau_1\frac{\partial}{\partial t})\nabla^2\theta=(n_1+\tau_0\frac{\partial}{\partial t})\dot{T}+\varepsilon(n_1+n_0\tau_0\frac{\partial}{\partial t})\dot{e}, \tag{23}$$

$$T=(1-a\nabla^2)\theta.$$

Where, $\alpha=1+\dfrac{\varepsilon_0\mu_0^2H_0^2}{\rho}$, $\beta=\dfrac{(\lambda+\mu)}{\rho c_0^2}$, $h_1=\beta+h_0$,

$$h_0=\frac{\mu_0H_0^2}{\rho c_0^2},\quad \varepsilon=\frac{\gamma^2T_0}{\rho^2C_E c_0^2},\quad a=\frac{a^*\omega^{*2}}{c_0^2}.$$

Also, the constitutive relations (16)-(19) reduces to

$$\sigma_{xx}=u_{,x}+(2\beta-1)v_{,y}-(1+\upsilon_0\frac{\partial}{\partial t})T, \tag{25}$$

$$\sigma_{yy}=(2\beta-1)u_{,x}+v_{,y}-(1+\upsilon_0\frac{\partial}{\partial t})T, \tag{26}$$

$$\sigma_{zz}=(2\beta-1)e-(1+\upsilon_0\frac{\partial}{\partial t})T, \tag{27}$$

$$\sigma_{xy}=(1-\beta)(u_{,y}+v_{,x}),\qquad \sigma_{xz}=\sigma_{yz}=0. \tag{28}$$

We define the displacement potentials $\Psi_1(x,y,t)$ and $\Psi_2(x,y,t)$ which relate to the displacement components u and v as

$$u = \Psi_{1,x} + \Psi_{2,y}, \qquad v = \Psi_{1,y} - \Psi_{2,x}. \tag{29}$$

3. NORMAL MODE ANALYSIS

The solution of the considered physical variables can be decomposed in terms of normal modes as the following form:

$$[u, v, e, T, \theta, \Psi_1^*, \Psi_2^*, \sigma_{ij}](x,y,t) = [u^*, v^*, e^*, T^*, \theta^*, \Psi_1^*, \Psi_2^*, \sigma_{ij}^*](x)\exp(\omega t + iby), \tag{30}$$

Where, the amplitudes of the field quantities are $[u^*, v^*, e^*, T^*, \theta^*, \Psi_1^*, \Psi_2^*, \sigma_{ij}^*](x)$, ω is the complex time constant, $i = \sqrt{-1}$ and b is the wave number in the $y-$direction.

Using Eqs. (24), (29) and (30), Eqs. (21)-(23) lead to

$$(\mathrm{D}^2 - b_1)\Psi_1^* - b_2\Psi_2^* + (b_3\mathrm{D}^2 - b_4)\theta^* = 0, \tag{31}$$

$$b_5\Psi_1^* + (\mathrm{D}^2 - b_6)\Psi_2^* = 0, \tag{32}$$

$$-b_7(\mathrm{D}^2 - b^2)\Psi_1^* + (b_8\mathrm{D}^2 - b_9)\theta^* = 0. \tag{33}$$

Also, the constitutive relations (25) - (28) becomes

$$\sigma_{xx}^* = \mathrm{D}u^* + ib(2\beta - 1)v^* - \omega_1 T^*, \tag{34}$$

$$\sigma_{yy}^* = (2\beta - 1)\mathrm{D}u^* + ibv^* - \omega_1 T^*, \tag{35}$$

$$\sigma_{zz}^* = (2\beta - 1)(\mathrm{D}^2 - b^2)\Psi_1^* - \omega_1 T^*, \tag{36}$$

$$\sigma_{xy}^* = (1-\beta)(ibu^* + \mathrm{D}v^*), \quad \sigma_{xz}^* = \sigma_{yz}^* = 0. \tag{37}$$

Where

$$\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}x}, \qquad b_1 = b^2 + \frac{\alpha\omega^2 - \Omega^2}{(1-\beta+h_1)}, \qquad b_2 = \frac{2\Omega\omega}{(1-\beta+h_1)}, \qquad \omega_1 = 1 + \upsilon_0\omega,$$

$$b_3 = \frac{a\omega_1}{(1-\beta+h_1)},$$

$$b_4 = \frac{\omega_1(1+ab^2)}{(1-\beta+h_1)}, \qquad b_5 = \frac{2\Omega\omega}{(-1\beta}, \qquad b_6 = b^2 + \frac{\alpha\omega^2-\Omega^2}{(1-\beta)},$$

$$b_7 = \varepsilon\omega(n_1+n_0\tau_0\omega),$$

$$b_8 = (n^* + t_1\omega) + a\omega(n_1+\tau_0\omega), \qquad b_9 = b^2(n^*+t_1\omega)+\omega(n_1+\tau_0\omega)(1+ab^2).$$

Eliminating $\theta^*(x)$ and $\Psi_2^*(x)$ between Eqs. (31)–(33), we get the following sixth order ordinary differential equation satisfied with $\Psi_1^*(x)$

$$[\mathrm{D}^6 - A_1\mathrm{D}^4 + A_2\mathrm{D}^2 - A_3]\Psi_1^*(x) = 0. \tag{38}$$

Where,

$$A_1 = \frac{b_6b_8+b_9+b_1b_8+b_3b_7(b^2+b_6)+b_4b_7}{(b_3b_7+b_8)},$$

$$A_2 = \frac{(b_1+b_6)b_9+(b_1b_6+b_2b_5)b_8+b_3b_6b_7b^2+b_4b_7(b^2+b_6)}{(b_3b_7+b_8)},$$

$$A_3 = \frac{(b_1b_6+b_2b_5)b_9+b_4b_6b_7b^2}{(b_3b_7+b_8)}.$$

Equation (38) can be factored as

$$(\mathrm{D}^2-k_1^2)(\mathrm{D}^2-k_2^2)(\mathrm{D}^2-k_3^2)\Psi_1^*(x) = 0, \tag{39}$$

Where $k_n^2 (n=1,2,3)$ are the roots of the characteristic equation of Eq. (38).

The solution of Eq. (38), which is bounded as $x \to \infty$, is given by

$$\Psi_1^*(x) = \sum_{n=1}^{3} M_n e^{-k_n x}. \tag{40}$$

In a similar manner, we get

$$\Psi_2^*(x) = \sum_{\mathrm{n}=1}^{3} H_{1n} M_{\mathrm{n}} e^{-k_n x}. \tag{41}$$

$$\theta^*(x) = \sum_{n=1}^{3} H_{2n} M_{\mathrm{n}} e^{-k_n x}. \tag{42}$$

Where $M_{\mathrm{n}}(n=1,2,3)$ are some parameters and $H_{1n}=\dfrac{b_5}{(b_6-k_n^2)}$, $H_{2n}=\dfrac{b_7(k_n^2-b^2)}{(b_8k_n^2-b_9)}$. Substituting from Eqs. (40)–(42) and (30) in Eqs. (29), (24), (34)-(37) respectively, we get

$$u^*(x)=\sum_{n=1}^{3}L_{1n}M_n\, e^{-k_nx}, \tag{43}$$

$$v^*(x)=\sum_{n=1}^{3}L_{2n}M_n\, e^{-k_nx}, \tag{44}$$

$$T^*(x)=\sum_{n=1}^{3}H_{3n}M_n\, e^{-k_nx}, \tag{45}$$

$$\sigma_{xx}^*=\sum_{n=1}^{3}L_{3n}M_n\, e^{-k_nx}, \tag{46}$$

$$\sigma_{yy}^*=\sum_{n=1}^{3}L_{4n}M_n\, e^{-k_nx}, \tag{47}$$

$$\sigma_{zz}^*=\sum_{n=1}^{3}L_{5n}M_n\, e^{-k_nx}, \tag{48}$$

$$\sigma_{xy}^*=\sum_{n=1}^{3}L_{6n}M_n\, e^{-k_nx}. \tag{49}$$

Where, $L_{1n}=ibH_{1n}-k_n$, $L_{2n}=ib+k_nH_{1n}$, $H_{3n}=[1-a(k_n^2-b^2)]H_{2n}$,

$L_{3n}=ib(2\beta-1)\mathrm{L}_{2\mathrm{n}}-k_nL_{1n}-\omega_1H_{3n}$,

$L_{4n}=ib\mathrm{L}_{2\mathrm{n}}-(2\beta-1)k_nL_{1n}-\omega_1H_{3n}$,

$L_{5n}=(2\beta-1)(k_n^2-b^2)-\omega_1H_{3n}$, $L_{6n}=(1-\beta)(\mathrm{ib}L_{1n}-k_n\mathrm{L}_{2\mathrm{n}})$.

The parameters $M_{\mathrm{n}}(n=1,2,3)$ have to be determined such that the boundary conditions on the surface $\mathrm{x}=0$ take the form

$$T(0,y,t)=f(0,y,t)=f^*\exp(\omega t+iby),\quad \sigma_{xx}(0,y,t)=\sigma_{xy}(0,y,t)=0. \tag{50}$$

Where $f(y,t)$ is an arbitrary function of y,t, and f^* is the magnitude of the constant temperature applied to the boundary.

Using the expressions of the variables considered into the above boundary conditions (50), we can obtain the following equations satisfied by the parameters $M_n (n=1,2,3)$

$$\sum_{n=1}^{3} H_{3n} M_n = f^*, \qquad \sum_{n=1}^{3} L_{3n} M_n = 0, \qquad \sum_{n=1}^{3} L_{6n} M_n = 0. \tag{51}$$

Solving Eqs. (51), we get the parameters $M_n (n=1,2,3)$ defined as follows:

$$M_1 = \frac{\Delta_1}{\Delta}, \qquad M_2 = \frac{\Delta_2}{\Delta}, \qquad M_3 = \frac{\Delta_3}{\Delta}. \tag{52}$$

Where,

$$\Delta = H_{31}[R_{32}R_{63} - R_{33}R_{62}] - H_{32}[R_{31}R_{63} - R_{33}R_{61}] + H_{33}[R_{31}R_{62} - R_{32}R_{61}],$$

$$\Delta_1 = f^*[R_{32}R_{63} - R_{33}R_{62}], \qquad \Delta_2 = -f^*[R_{31}R_{63} - R_{33}R_{61}],$$

$$\Delta_3 = f^*[R_{31}R_{62} - R_{32}R_{61}].$$

4. Particular cases

1. Neglecting the magnetic field (i.e. $H_0 = 0$) in the above equations, the expressions for the displacement components, force stresses, conductive temperature and thermodynamic temperature distribution in a rotating generalized thermoelastic medium with two temperature can be obtained.

2. The expressions for the displacement components, force stresses and temperature distribution in a rotating generalized magneto-thermoelastic medium can be obtained
from the above equations by taking $a=0$ ($a=0$ indicates one type temperature).

3. Neglecting the angular velocity (i.e. $\Omega = 0$) in the above equations, one can obtain the displacement components, force stresses, conductive temperature and temperature distribution in a non-rotating generalized magneto-thermoelastic medium with two temperature.

After substituting $\Omega = 0$ in Eqs. (5), (13)-(14) and use Eqs. (20), (29) and (30), it can be reached that:

$$(\mathrm{D}^2 - b_1')\Psi_1^* + (b_3\mathrm{D}^2 - b_4)\theta^* = 0, \tag{53}$$

$$(\mathrm{D}^2 - m^2)\Psi_2^* = 0, \tag{54}$$

$$-b_7(\mathrm{D}^2 - b^2)\Psi_1^* + (b_8\mathrm{D}^2 - b_9)\theta^* = 0. \tag{55}$$

Eliminating $\theta^*(x)$ and $\Psi_1^*(x)$ in Eqs. (53) and (55), we get the following fourth order ordinary differential equations for $\theta^*(x)$ and $\Psi_1^*(x)$:

$$[\mathrm{D}^4 - A\mathrm{D}^2 + B]\{\Psi_1^*(x), \theta^*(x)\} = 0. \tag{56}$$

Equation (56) can be factored as

$$(\mathrm{D}^2 - k_1^2)(\mathrm{D}^2 - k_2^2)\{\Psi_1^*(x), \theta^*(x)\} = 0. \tag{57}$$

Where $k_n^2 (n = 1, 2)$ are the roots of the characteristic equation of Eq. (56), $A = \dfrac{b_1'b_8 + b_9 + b_7(b_4 + b_3 b^2)}{(b_3 b_7 + b_8)}$, $B = \dfrac{b_1'b_9 + b_4 b_7 b^2}{(b_3 b_7 + b_8)}$,

$b_1' = b^2 + \dfrac{\alpha\omega^2}{(1 - \beta + h_1)}$, $m^2 = b^2 + \dfrac{\alpha\omega^2}{(1 - \beta)}$.

The solution of Eqs. (56) and (54), take the form

$$\Psi_1^*(x) = \sum_{n=1}^{2} Z_n\, e^{-k_n x}, \tag{58}$$

$$\theta^*(x) = \sum_{n=1}^{2} G_{1n} Z_n\, e^{-k_n x}, \tag{59}$$

$$\Psi_2^*(x) = Z_3\, e^{-mx}. \tag{60}$$

Where $Z_n (n = 1, 2, 3)$ are some parameters and $G_{1n} = \dfrac{k_n^2 - b_1'}{b_4 - b_3 k_n^2}$.

Using Eqs. (24)-(30), (58)-(60), we get the expressions for displacement components, force stresses, conductive temperature and temperature distribution in a non-rotating generalized magneto-thermoelastic medium with two temperature as follows:

$$u^*(x) = -\sum_{n=1}^{2} k_n Z_n e^{-k_n x} + ibZ_3 e^{-mx}, \tag{61}$$

$$v^*(x) = ib\sum_{n=1}^{2} Z_n e^{-k_n x} + m Z_3 e^{-mx}, \tag{62}$$

$$\theta^*(x) = \sum_{n=1}^{2} G_{1n} Z_n e^{-k_n x}, \tag{63}$$

$$T^*(x) = \sum_{n=1}^{2} G_{2n} Z_n e^{-k_n x}, \tag{64}$$

$$\sigma_{xx}^*(x) = \sum_{n=1}^{2} G_{3n} Z_n e^{-k_n x} + s_1 Z_3 e^{-mx}, \tag{65}$$

$$\sigma_{yy}^*(x) = \sum_{n=1}^{2} G_{4n} Z_n e^{-k_n x} - s_1 Z_3 e^{-mx}, \tag{66}$$

$$\sigma_{zz}^*(x) = \sum_{n=1}^{2} G_{5n} Z_n e^{-k_n x}, \tag{67}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{2} G_{6n} Z_n e^{-k_n x} + s_2 Z_3 e^{-mx}. \tag{68}$$

Where,

$$G_{2n} = [1 - a(k_n^2 - b^2)]G_{1n}, \qquad G_{3n} = k_n^2 - b^2(2\beta - 1) - \omega_1 G_{2n},$$

$$G_{4n} = (2\beta - 1)k_n^2 - b^2 - \omega_1 G_{2n}, \qquad G_{5n} = (2\beta - 1)(k_n^2 - b^2) - \omega_1 G_{2n},$$

$$G_{6n} = 2ib(\beta - 1)k_n, \quad s_1 = 2ibm(\beta - 1), \quad s_2 = (m^2 + b^2)(\beta - 1).$$

Applying the boundary conditions (50) at the surface $x = 0$, a system of three equations is obtained. After solving this system, the coefficients $Z_n (n = 1, 2, 3)$ can be defined as follows:

$$Z_1 = \frac{\Delta_1'}{\Delta'}, \qquad Z_2 = \frac{\Delta_2'}{\Delta'}, \qquad Z_3 = \frac{\Delta_3'}{\Delta'}. \tag{69}$$

Where,

$$\Delta' = G_{21}[G_{32}s_2 - G_{62}s_1] - G_{22}[G_{31}s_2 - G_{61}s_1], \qquad \Delta_1' = f^*[G_{32}s_2 - G_{62}s_1],$$

$$\Delta_2' = -f^*[G_{31}s_2 - G_{61}s_1], \qquad \Delta_3' = f^*[G_{31}G_{62} - G_{32}G_{61}].$$

5. Special cases of thermoelastic theory

Equations (3) and (5) are the field equations of the generalized linear magneto-thermoelasticity for a rotating media, applicable to the coupled theory, four generalizations, as follows:

Theory	n^*	n_1	n_0	t_1	τ_0	υ_0
The coupled theory (CT)	1	1	>0	0	0	0
Lord-Shulman theory (L-S)	1	1	1	0	>0	0
Green-Lindsay theory (G-L)	1	1	0	0	>0	>0
Green-Naghdi without energy dissipation (G-N II)	1	0	1	0	1	0
Chandrasekaraiah-Tzou theory (DPL)	1	1	1	>0	>0	0

6. Numerical results and discussions

Copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as follows:

$\lambda = 7.76 \times 10^{10}\ N.m^{-2}$, $\qquad \mu = 3.86 \times 10^{10}\ kg.m^{-1}.s^{-2}$,

$K = 386\, w.m^{-1}.k^{-1}$,

$\alpha_t = 1.78 \times 10^{-5} k^{-1}$, $\qquad \rho = 8954\, kg.m^{-3}$, $\qquad C_E = 383.1\, J.kg^{-1}.k^{-1}$,

$T_0 = 293\, K$.

The comparisons were carried out for

$y = 0.1,\quad f^{*} = 1,\quad \omega = \omega_0 + i\,\xi,\quad \omega_0 = 0.7\quad \xi = 0.1\quad b = 0.5$

$t_1 = 0.6,\quad \tau_0 = 0.8,\quad \upsilon_0 = 1.1,\quad a = 0.1,\quad \Omega = 0.1,\quad H_0 = 10^8.$

The computations were carried out for a value of time $t = 0.7$. The numerical data, outlined above, were used for the distribution of the real part of the displacement components u and v, the conductive temperature θ, the thermodynamic temperature T and the stress components σ_{xx} and σ_{xy} for the problem under consideration. All the considered variables depend not only on the variables t, x and y, but also depend on the thermal relaxation times t_1, τ_0 and υ_0. The results are shown in Figs. 1-9. The graphs show the six curves predicted by three different theories of thermoelasticity (G-L, G-N II, and DPL). In these figures, the solid lines represent the solution in the generalized G-L theory, the dashed lines represent the solution using G-N II model and the dashed-dotted lines represent the solution in the context of the DPL model. Here all the variables are taken in non-dimensional forms.

Figures 1-4 show comparisons among the considered variables in the absence and presence of the magnetic field (i.e. $H_0 = 0, 10^8$) for $\Omega = 0.1$ and $a = 0.1$.

Figure 1 shows that the distribution of the vertical displacement v always begins from negative values. In the context of the three theories, the values of the vertical displacement v decrease in the beginning to a minimum value in the range $0 \le x \le 1.2$, then increase in the range $1.2 \le x \le 8$, and also move in a wave propagation. It is also clear that the magnetic field acts to increase the magnitude of the real part of v and the values of v based on the G-N II model are large compared to those based on the G-L theory while they are small compared to those based on the DPL model.

Figures 2 and 3 demonstrate that the distribution of the conductive temperature θ and the thermodynamic temperature T begins from a positive value and satisfies the boundary conditions at $x=0$. In the context of the three theories and in the absence and presence of a magnetic field (i.e. $H_0=0,10^8$), θ and T decrease in the range $0\le x\le 8$. The values of θ and T converge to zero with increasing the distance x at $x\ge 8$. It is also obvious from this figure that the magnetic field has no great effect on the distribution of the conductive temperature θ and the thermodynamic temperature T. Also, the values of both θ and T in the context of the DPL model are higher than those in the context of the G-L theory while they are lower than those in the context of the G-N II model.

Figure 4 shows the distribution of the stress component σ_{xy} and demonstrates that it reaches a zero value and satisfies the boundary conditions at $x=0$. In the context of the three theories, the values of σ_{xy} increase in the beginning to a maximum value in the range $0\le x\le 1.7$, then decrease in the range $1.7\le x\le 8$. It is clear that the magnetic field acts to decrease the magnitude of the real part of σ_{xy}. This is mainly due to the fact of magnetic field corresponds to the term signifying positive force that tend to accelerate the model particles. The values of σ_{xy} based on the G-N II model are large compared to those in the context of the DPL model while they are small compared to those in the context of the G-L theory.

Figures 5-7 show comparisons among the considered variables for different values of $\Omega\,(\Omega = 0, 0.1)$ in the presence of a magnetic field $H_0 = 10^8$ and $a = 0.1$.

Figure 5 depicts that the distribution of the horizontal displacement u always begins from negative values. In the context of the three theories, the values of u start with increasing to a maximum value in the range $0 \le x \le 1.2$, then decreasing in the range $1.2 \le x \le 8$ for $\Omega = 0.1$. However, in the context of the three theories, the values of u increases in the range $0 \le x \le 8$ for $\Omega = 0$. It also shows that the rotation acts to increase the magnitude of the real part of u.

Figure 6 depicts that the distribution of the thermodynamic temperature T always begins from a positive value and satisfies the boundary conditions at $x = 0$. In the context of the three theories, the values of the temperature T decrease in the range $0 \le x \le 8$ for $\Omega = 0, 0.1$.

Figure 7 demonstrates that the distribution of the stress component σ_{xx}, in the context of the three theories, begins from zero and satisfies the boundary conditions at $x = 0$ for $\Omega = 0, 0.1$. In the context of the three theories, the values of σ_{xx} start with decreasing to a minimum value in the range $0 \le x \le 1.2$, then increase in the range $1.2 \le x \le 8$ for $\Omega = 0, 0.1$. It is also clear that the rotation acts to decrease the magnitude of the real part of σ_{xx}.

Figures 8-9 show comparisons among the considered variables for two different values of the non-dimensional, two temperature parameter $a(a = 0, 0.1)$ where $a = 0$ indicates one-type temperature and $a = 0.1$ indicates two-type temperature in the presence of a magnetic field (i.e. $H_0 = 10^8$) and rotation (i.e. $\Omega = 0.1$).

Figure 8 shows the variation of the vertical displacement v for $a = 0, 0.1$. In this figure, a significant difference in the vertical displacement v is noticed for different values of the non-dimensional two temperature parameter a. Also, this figure shows that the magnitude of v for $a = 0.1$ is higher than that of $a = 0$.

Figure 9 gives the variation of the conductive temperature θ versus distance x in the case of one-type temperature as well as two-type temperature. For both of the cases, it is observed that conductive temperature decreases with the increase of distance and finally goes to zero.

3D curves 10-12 are representing the relation between the physical quantities and both components of distance, in the presence of the magnetic field $H_0 = 10^8$ and the rotation effect $\Omega = 0.1$ in a generalized thermoelastic medium with two temperature ($a = 0.1$), in the context of the dual-phase-lag model (DPL). These figures are very important to study the dependence of these physical quantities on the vertical component of distance. The curves obtained are highly depending on the vertical distance and all the physical quantities are moving in wave propagation.

7. Conclusion

By comparing the figures that were obtained for the three thermoelastic theories, important phenomena are observed:

1. The values of all physical quantities converge to zero with increasing distance x,

and all functions are continuous.

2. All physical quantities to satisfy the boundary conditions.
3. The phenomenon of finite speeds of propagation is manifested in all
figures..
4. The rotation and magnetic field have important roles in the distribution
of the field quantities except the temperature.
5. Analytical solutions based upon normal mode analysis of the
thermoelastic problem in solids have been developed and utilized.

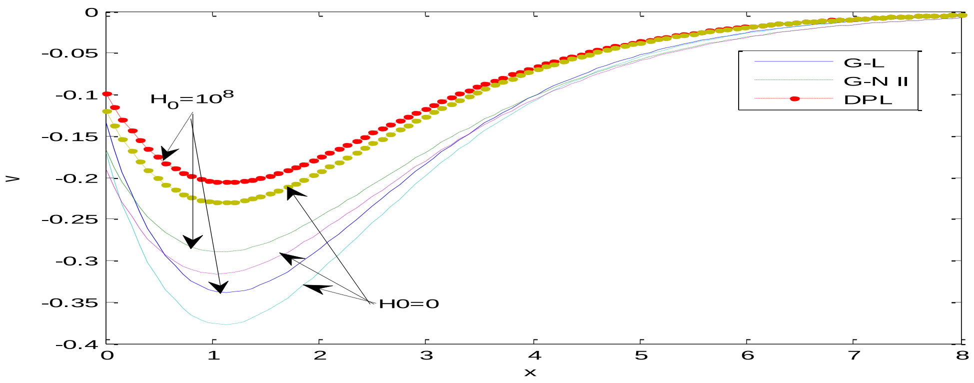

Fig. 1 Vertical displacement distribution v in the absence and presence of magnetic field

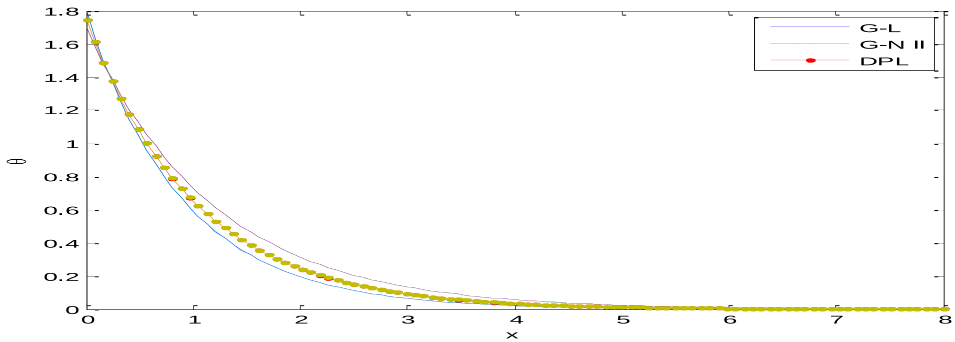

Fig. 2 Conductive temperature distribution θ in the absence and presence of magnetic field

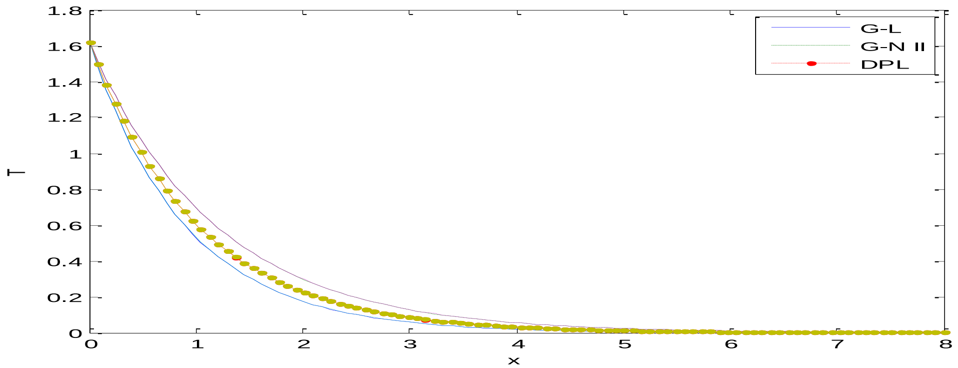

Fig. 3 Thermodynamic temperature distribution T in the absence and presence of magnetic field

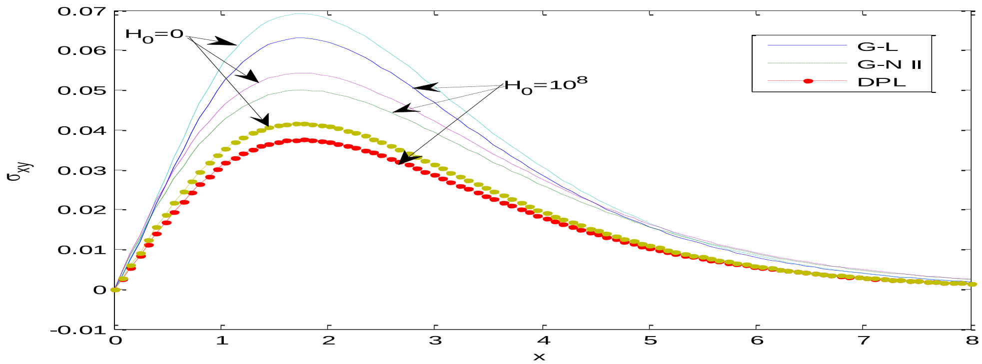

Fig. 4 Distribution of stress component σ_{xy} in the absence and presence of magnetic field

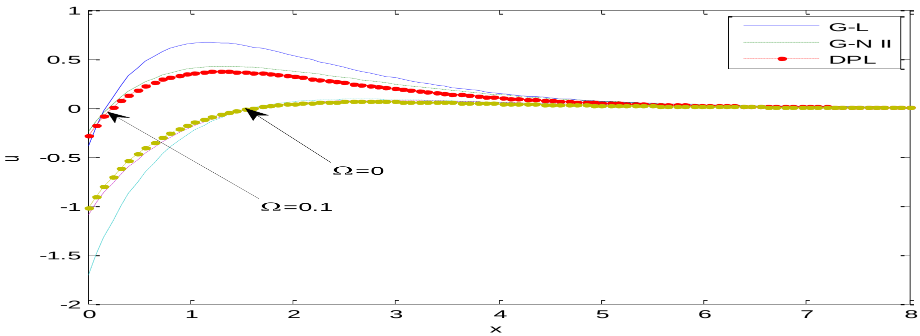

Fig. 5 Horizontal displacement distribution u in the absence and presence of rotation

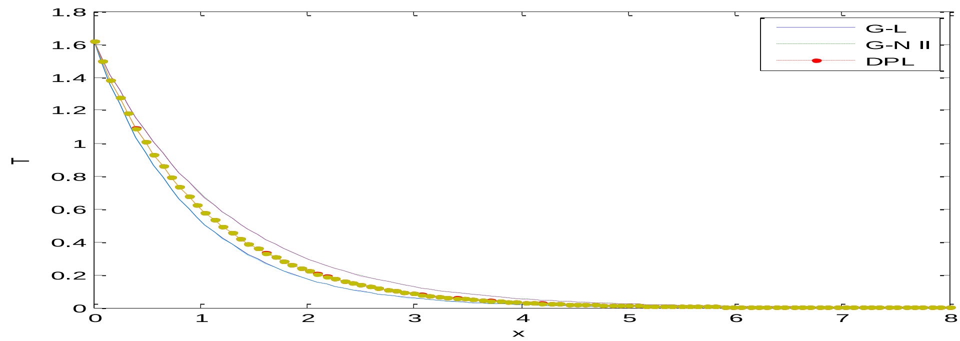

Fig. 6 Thermodynamic temperature distribution T in the absence and presence of rotation

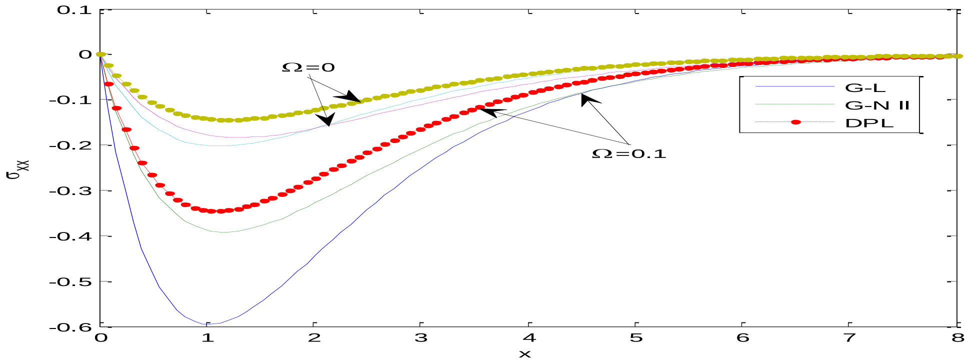

Fig.7 Distribution of stress component σ_{xx} in the absence and presence of rotation

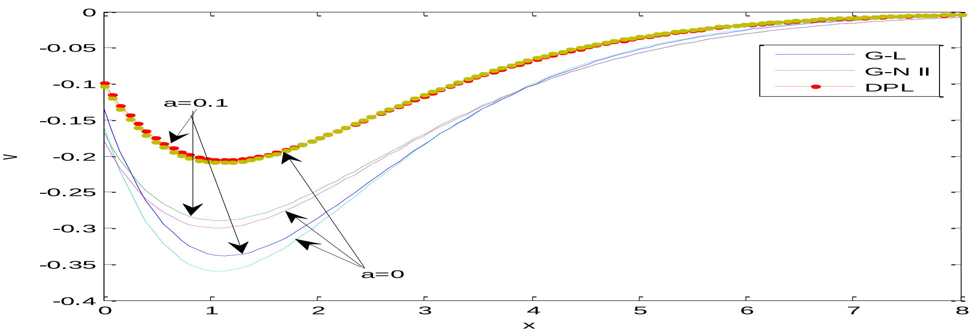

Fig.8 Vertical displacement distribution v for two different values of a two temperature parameter $a = 0, 0.1$

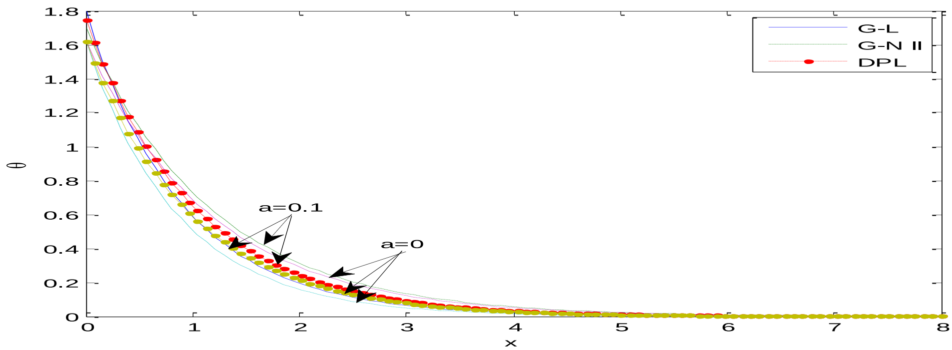

Fig. 9 Conductive temperature distribution θ for two different values of a two temperature parameter $a = 0, 0.1$

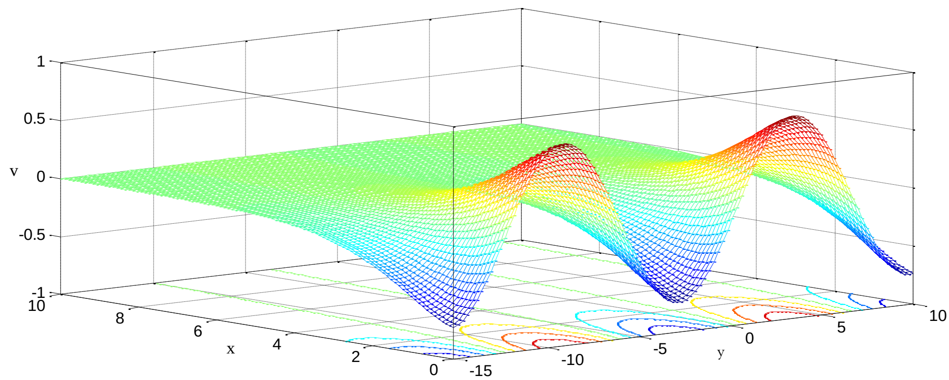

Fig. 10 (3D) Distribution of the displacement component v against both components

of distance based on DPL model at $\Omega = 0.1$, $a = 0.1$ and $H_0 = 10^8$.

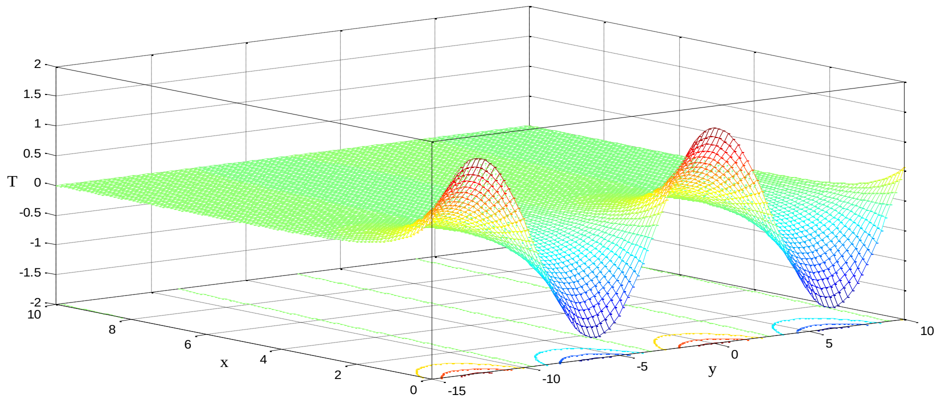

Fig. 11 (3D) Thermodynamic temperature distribution T against both components of distance based on DPL model at $\Omega = 0.1,\ a = 0.1$ and $H_0 = 10^8$.

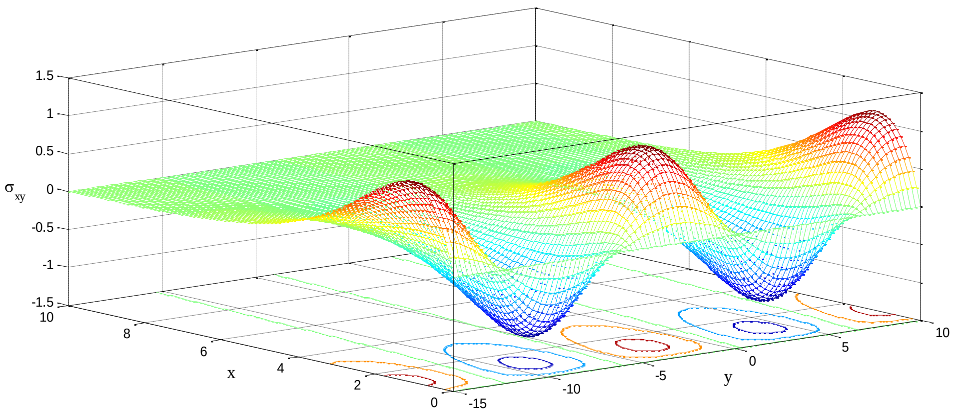

Fig. 12 (3D) Distribution of the stress component σ_{xy} against both components of distance based on DPL model at $\Omega = 0.1,\ a = 0.1$ and $H_0 = 10^8$.

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