الحدية في الفضاءات الضبابية الحدسية B

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الملخص:

الحديه في الفضاءات الضبابية الحدسيه وتم اثبات βقدمنا في هذا البحث مفهوم المجموعات العديد مم خصائصها. تعتبر الفضاءات الحدةيه مادة مفيدة جدا للبحاثين في هذا المجال

β boundary in Intuitionistic fuzzy topological spaces

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Abstract:

In this paper the concept of Intuitionistic fuzzy β boundary is introduced and several of its properties are investigated. Intuitionistic fuzzy set is very useful in providing sufficient material for researchers to utilize these concepts fruitfully.

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Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topology spaces, Intuitionistic fuzzy β open set, Intuitionistic fuzzy βclosed set, Intuitionistic fuzzy β boundary**,**

Introduction:

After the introduction of fuzzy sets by Zadeh [9], the fuzzy concept has invaded almost all branches of mathematics.The concept of fuzzy topological spaces was introduced and developed by Chang [4]. Atanassov [1,2] introduced the notion of intuitionistic fuzzy sets, Coker [5] introduced the intuitionistic fuzzy topological spaces. Using the notion of intutionistic fuzzy β set, we also define the concept of intuitionistic fuzzy βboundary. We extend this study further in the last paper and give many properties, characterizations and examples pertaining to the generalized notion.

The aim of this paper is to conclude several properties of Intuitionistic fuzzy β boundary and give some examples about it.

1- Preliminaries:

Definition 2.1. [1] Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS for short) A in X is defined by

$$A = \{\langle x, \mu_A(x), v_A(x)\rangle \mid x \in X\}.$$

Where the functions $\mu_A: X \to [0,1]$ and $v_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $v_A(x)$) of each element $x \in X$ to the set A, respectively. And $0 \leq \mu_A(x) + v_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X) the set of all intuitionistic fuzzy sets in X.

Definition 2.2. [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), v_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), v_B(x)\rangle \mid x \in X\}$. Then

1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$,
2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
3) $A^c = 1 - A = \{\langle x, v_A(x), \mu_A(x)\rangle \mid x \in X\}$,
4) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), v_A(x) \vee v_B(x)\rangle \mid x \in X\}$,
5) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), v_A(x) \wedge v_B(x)\rangle \mid x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, v_A\rangle$ instead of $A = \{\langle x, \mu_A(x), v_A(x)\rangle \mid x \in X\}$.

Definition 2.3, [1] $0 = \{\langle x, 0, 1\rangle : x \in X\}$ and $1 = \{\langle x, 1, 0\rangle : x \in X\}$

Definition 2.4.[4] An intuitionistic fuzzy topological space (IFTS for short) on X is a family τ of IFSs in X which satisfies the following properties:

1) $0, 1 \in \tau$.
2) If $A_1, A_2 \in \tau$, then $A_1 \cap A_2 \in \tau$.
3) If $A_i \in \tau$ for each i, then $\cup A_i \in \tau$.

The pair (X, τ) is called an intuitionistic fuzzy topological space.

Definition 2.5. [4] Each member of τ is called an intuitionistic fuzzy open set (IFOS for short) in X.

Definition 2.6. [3] The complement A^c of an IFOS A in X is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.7. [5] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A, v_A\rangle\}$ be an IFS in X. Then the intuitionistic fuzzy interior is defined by

$$int(A) = \cup \{U \mid U \text{ is an IFOS in } X \text{ and } U \subseteq A\},$$

and an intuitionistic fuzzy closure is defined by

$$cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Now, it is important to introduce the following propositions.

Proposition 2.8. [2] Let (X, τ) be an IFTS and A, B be IFSs in X. Then the following properties hold:

1) $int(A) \subseteq A$,
2) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$,
3) $int(int(A)) = int(A)$,
4) $int(A \cap B) = int(A) \cap int(B)$,
5) $int(1) = 1$ and $int(0) = 0$.
6) $A \subseteq cl(A)$,
7) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$,
8) $cl(cl(A)) = cl(A)$,
9) $cl(A \cup B) = cl(A) \cup cl(B)$,
10) $cl(0) = 0$ and $cl(1) = 1$.
11) $cl(A^c) = (int(A))^c$,
12) $int(A^c) = (cl(A))^c$.

Definition 2.9. [6] Let (X, τ) be an IFTS and A be an IFS in X. Then the intuitionistic fuzzy boundary of A is defined as $IBd(A) = cl(A) \cap cl(A^c)$. $IBd(A)$ is a intuitionistic fuzzy closed set (IFCS).

3- Intuitionistic fuzzy βboundary

Definition 3.1. [3, 8] An IFS A in an IFTS X is called an intuitionistic fuzzy βopen set (IFβOS) of X, if and only if $A \subseteq cl(int(cl(A)))$. The complement of an IFβOS A in X is called intuitionistic fuzzy βclosed (IFβCS) in X.

Example 3.2.

Let $X = \{a, b\}$ and

$$A = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.3}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right) \rangle,$$

$$B = \langle x, \left(\frac{a}{0.3}, \frac{b}{0.7}\right), \left(\frac{a}{0.7}, \frac{b}{0.3}\right) \rangle.$$

Then the family $\tau = \{0, 1, A\}$ is an IFT on X. Since $B \subseteq cl\left(int(cl(B))\right) = A^c$, B is an IFβOS in X.

Definition 3.3. [8] Let (X, τ) be an IFTS and $A = \{\langle x, \mu_A, \nu_A \rangle\}$ be an IFS in X.Then the intuitionisitic fuzzy βinterior and intuitionistic fuzzy βclosure of A are defined by

$$\beta int(A) = \cup \{U \mid U \text{ is an IF } \beta\text{OS in } X \text{ and } U \subseteq A\},$$
$$\beta cl(A) = \cap \{K \mid K \text{ is an IF } \beta\text{CS in } X \text{ and } A \subseteq K\}.$$

Remark 3.4. From definition we have:

1) $\beta int(A) \subseteq A \subseteq \beta cl(A)$.
2) If $A \subseteq B \Longrightarrow \beta int(A) \subseteq \beta int(B)$.
3) If $A \subseteq B \Longrightarrow \beta cl(A) \subseteq \beta cl(B)$.
4) If A IFβOS. Then $\beta int(A) = A$.
5) If A IFβCS. Then $\beta cl(A) = A$.

Example 3.5.

Let (X, τ) be an IFTS defined in example 3.2. $\beta int(B) = B$, $\beta cl(B) = B$

Proposition 3.6. Any IFOS is IFβOS.

Proof:

Let (X, τ) be an IFTS and A be an IFOS in X.Then $A = int(A)$.

$\because A \subseteq cl(A)$ …….. (i)

$\Longrightarrow int(A) \subseteq int(cl(A))$.Then we have $\Longrightarrow A \subseteq int(cl(A))$

From (i) $A \subseteq cl(A) \subseteq cl\left(int\left(cl(A)\right)\right)$. Then A is an IFβOS in X.

Remark 3.7. From above proposition:

1) Let (X, τ) be an IFTS and A be IFS. Then $int(A) \subseteq \beta int(A)$.
2) Let (X, τ) be an IFTS and A be IFS. Then $cl(A) \subseteq \beta cl(A)$
3) The converse of above proposition is not true.

Example 3.8.

Let (X, τ) be an IFTS defined in example 3.2. B is IFβOS in X. But is not IFOS in X.

Theorem 3.9. For IFSs $A \; and \; B$ in an IFTS (X, τ). Then.

1) $\beta int(A \cap B) = \beta int(A) \cap \beta int(A)$.
2) $\beta cl(A \cup B) = \beta cl(A) \cup \beta cl(A)$.
3) $\beta cl(A \cap B) \subseteq \beta cl(A) \cap \beta cl(B)$.
4) $\beta cl\left(\beta cl(A)\right) = \beta cl(A)$
5) $\beta int(\beta int(A)) = \beta int(A)$.

6) $(\beta int(A))^c = \beta cl(A^c)$.
7) $(\beta cl(A))^c = \beta cl(A^c)$.

Proof:

1) Since $\beta int(A) \subseteq A,\ \ \beta int(B) \subseteq B$. Then $\beta int(A) \cap \beta int(B) \subseteq A \cap B$. But $\beta int(A) \cap \beta int(B)$ is an intuitionisitic fuzzy βopen set
$\Longrightarrow\ \beta int(A) \cap \beta int(B) \subseteq \beta int(A \cap B)$ ....(i)
Conversely, $A \cap B \subseteq A\ , A \cap B \subseteq B$
$\Longrightarrow \beta int(A \cap B)\ \subseteq \beta int(A) \cap \beta int(B)$ .... (ii)
From (i) and (ii) we have, $\beta int(A \cap B) = \beta int(A) \cap \beta int(B)$.
2) Since $A \subseteq A \cup B\ \ , B \subseteq A \cup B$. Then we have $\beta cl(A) \subseteq \beta cl(A \cup B)$, and $\beta cl(B) \subseteq \beta cl(A \cup B)$
$\Longrightarrow\ \beta cl(A) \cup \beta cl(B) \subseteq \beta cl(A \cup B)$.........(i)
Since, $A \subseteq \beta cl(A)\ , B \subseteq \beta cl(B)$. Hence
$\beta cl(A \cup B) \subseteq\ \beta cl(A) \cup \beta cl(B)$ ....(ii)
From (i) and (ii) we have, $\beta cl(A \cup B) =\ \beta cl(A) \cup \beta cl(B)$.
3) Since $A \cap B \subseteq A,\ A \cap B \subseteq B$
$\Longrightarrow \beta cl(A \cap B)\ \subseteq\ \beta cl(A)$ and $\beta cl(A \cap B)\ \subseteq\ \beta cl(B)$
Hence $\beta cl(A \cap B)\ \subseteq \beta cl(A) \cap \beta cl(B)$.
4) From definition $\beta cl(A)$ is a intuitionisitic fuzzy βclosed set. Then we have $\beta cl\big(\beta cl(A)\big) = \beta cl(A)$.
5) From definition $\beta int(A)$ is a intuitionisitic fuzzy βopen set. Then we have $\beta int\big(\beta int(A)\big) = \beta int(A)$.
(7)-(8) proofs are straight forward.

Example 3.10.

Let (X, τ) be an IFTS , $\tau = \{0,1, A, B\}$. Such that

$A = \langle x, \left(\frac{a}{0.7}, \frac{b}{0.7}\right), \left(\frac{a}{0.3}, \frac{b}{0.2}\right)\rangle\ ,\ B = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\rangle$.

$C = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.1}\right), \left(\frac{a}{0.5}, \frac{b}{0.7}\right)\rangle\ ,\ D = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.5}\right), \left(\frac{a}{0.7}, \frac{b}{0.4}\right)\rangle$.

$\beta cl(C) = 1\ , \beta cl(D) = 1$ and $\beta cl(C \cap D) = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.1}\right), \left(\frac{a}{0.7}, \frac{b}{0.7}\right)\rangle$

$\beta cl(C \cap D) \neq \beta cl(C) \cap \beta cl(D)$.

Definition 3.11. [6] Let A be IFS in an IFTS (X, τ). Then the intuitionistic fuzzy boundary of A is defined as $IBd(A) = cl(A) \cap cl(A^c)$. $IBd(A)$ is an intuitionistic fuzzy closed set (IFCS).

Definition 3.12. Let A be IFS in an IFTS (X,τ). Then the intuitionistic fuzzy βboundary of A is defined as $I\beta Bd(A) = \beta cl(A) \cap \beta cl(A^c)$. $I\beta Bd(A)$ is an intuitionistic fuzzy βclosed set (IFβCS).

Example 3.13.
Let (X,τ) be an IFTS defined in example 3.2 ,$\beta cl(B) = B, \beta cl(B^c) = B^c$. Then $I\beta Bd(B) = A$.

Remark 3.14. In classical topology, for an arbitrary set A of topological space X, $A \cup Bd(A) = cl(A)$. But $A \cup I\beta Bd(A) \subseteq \beta cl(A)$, for an arbitrary intuitionistic fuzzy set A in an IFTS (X,τ), where the equality may not hold.

Example 3.15.
Let (X,τ) be an IFTS defined in example 3.10. $C \cup I\beta Bd(C) = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.7}\right), \left(\frac{a}{0.5}, \frac{b}{0.1}\right)\rangle \neq \beta cl(C) = 1$

Proposition 3.16. Let A and B be IFSs in an IFTS (X,τ). Then the following conditions hold:

1) $I\beta Bd(A) \subseteq IBd(A)$.
2) $\beta cl(I\beta Bd(A)) \subseteq IBd(A)$
3) $I\beta Bd(A) = I\beta Bd(A^c)$.
4) If A is IFβCS, then $I\beta Bd(A) \subseteq A$.
5) If A is IFβOS, then $I\beta Bd(A) \subseteq A^c$.
6) $(I\beta Bd(A))^c = \beta int(A) \cup \beta int(A^c)$.

Proof:

1) Since $\beta cl(A) \subseteq cl(A)$ and $\beta cl(A^c) \subseteq cl(A^c)$. Then $I\beta Bd(A) = \beta cl(A) \cap \beta cl(A^c) \subseteq cl(A) \cap cl(A^c) = IBd(A)$. Hence, $I\beta Bd(A) \subseteq IBd(A)$.
2) $$\begin{aligned} \beta cl(I\beta Bd(A) &= \beta cl(\beta cl(A) \cap \beta cl(A)^c \\ &\subseteq \beta cl\big(\beta cl(A)\big) \cap \beta cl\big(\beta cl(A^c)\big) \\ &= \beta cl(A) \cap \beta cl(A^c) = I\beta Bd(A) \subseteq IBd(A). \end{aligned}$$
3) $$\begin{aligned} I\beta Bd(A) &= \beta cl(A) \cap \beta cl(A^c) \\ &= \beta cl(A^c)^c \cap \beta cl(A^c) \\ &= \beta cl(A^c) \cap \beta cl(A^c)^c = I\beta Bd(A^c). \end{aligned}$$
4) Let A be an IFβCS.$Then$
$I\beta Bd(A) = \beta cl(A) \cap \beta cl(A)^c \subseteq \beta cl(A) = A$
$\Longrightarrow I\beta Bd(A) \subseteq A$.

5) If A is IFβOS, then A^c is IFβCS. By (3) and (4) we have $I\beta Bd(A) = I\beta Bd(A^c) and\ I\beta Bd(A^c) \subseteq A^c \Longrightarrow I\beta Bd(A) \subseteq A^c$.
6) $\left(I\beta Bd(A)\right)^c = \left(\beta cl(A) \cap \beta cl(A^c)\right)^c = \left(\beta cl(A)\right)^c \cup \left(\beta cl(A^c)\right)^c$
$= \beta int(A^c) \cup \beta int(A)$
Then we have $(I\beta Bd(A))^c = \beta int(A) \cup \beta int(A^c)$.

Example 3.17.

Let (X, τ) be an IFTS defined in example 3.2. $I\beta Bd(B) = A$, $IBd(B) = A^C\ and\ A \subseteq A^C$. $\beta cl\left(I\beta Bd(B)\right) = A \subseteq IBd(A) = A^C$.

Proposition 3.18. Let A be a intuitionistic fuzzy set in an IFTS (X, τ). Then,

1) $I\beta Bd(A) = \beta cl(A) \cap (\beta int(A))^c$.
2) $I\beta Bd(\beta int(A) \subseteq I\beta Bd(A)$.,
3) $\beta int(A) \subseteq A \cap (I\beta Bd(A))^c$.

Proof:

1) Since $I\beta Bd(A) = \beta cl(A) \cap \beta cl(A^c)$. But $\beta cl(A^c) = \left(\beta int(A)\right)^c$.
Then $I\beta Bd(A) = \beta cl(A) \cap \left(\beta int(A)\right)^c$
2) $I\beta Bd(\beta int(A) = \beta cl(\beta int(A)) \cap \beta cl(\beta int(A))^c$
$= \beta cl\left(\beta int(A)\right) \cap \beta cl(\beta cl(A)^c)$
$= \beta cl\left(\beta int(A)\right) \cap \beta cl(A)^c$
$\subseteq \beta cl(A) \cap \beta cl(A)^c = I\beta Bd(A)$.
3) Consider
$A \cap (I\beta Bd(A))^c = A \cap [\beta cl(A) \cap \beta cl(A)^c]^c$
$= A \cap [\beta int(A)^c \cup \beta int(A)]$
$= (A \cap \beta int(A)^c) \cup \left(A \cap \beta int(A)\right)$
$\supseteq \beta int(A)$

Example 3.19.

Let (X, τ) be an IFTS such that

$A = \langle x, \left(\frac{a}{0.7}, \frac{b}{0.7}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}\right)\rangle$, $B = \langle x, \left(\frac{a}{0.8}, \frac{b}{0.8}\right), \left(\frac{a}{0.1}, \frac{b}{0.2}\right)\rangle$.

$C = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.8}, \frac{b}{0.8}\right)\rangle, \tau = \{0,1, A, B\}$. Then $I\beta Bd(C) = C$
$but\ \beta int(C) = 0$. Then $I\beta Bd(\beta int(C) \neq I\beta Bd(C)$ and
$\beta int(C) \subseteq C \cap \left(I\beta Bd(C)\right)^c = C$

Theorem 3.20. Let A and B be IFSs in an IFTS (X, τ). Then

$$I\beta Bd(A\cup B)\subseteq I\beta Bd(A)\cup I\beta Bd(B).$$

Proof:

From Theorem 3.9, we have

$I\beta Bd(A\cup B)=\beta cl(A\cup B)\cap\beta cl(A\cup B)^c$

$=\beta cl(A\cup B)\cap\beta cl((A^c)\cap(B^c))$

$\subseteq(\beta cl(A)\cup\beta cl(B))\cap(\beta cl(A^c)\cap\beta cl(B^c)).$

$=\beta cl(A)\cap(\beta cl(A^c)\cap\beta cl(B^c))\cup(\beta cl(B)\cap$
$(\beta cl(A^c)\cap\beta cl(B^c)).$

$=(\beta IBd(A)\cap(\beta cl(A^c))\cup(\beta IBd(B)\cap(\beta cl(A^c))$

$\subseteq\beta IBd(A)\cup\beta IBd(B).$

$\Longrightarrow I\beta Bd(A\cup B)\subseteq\beta IBd(A)\cup\beta IBd(B).$

Example 3.21.

Let (X,τ) be an IFTS defined in example 3.14. $I\beta Bd(C\cup B)=B^C$, but $\beta IBd(C)\cup\beta IBd(B)=C$. Then

$I\beta Bd(C\cup B)\neq\beta IBd(C)\cup\beta IBd(B)$

Theorem 3.22. Let A and B be IFSs in an IFTS (X,τ). Then

$$I\beta Bd(A\cap B)\subseteq(I\beta Bd(A)\cap\beta cl(B))\cup(I\beta Bd(B)\cap\beta cl(A)).$$

Proof:

$I\beta Bd(A\cap B)=\beta cl(A\cap B)\cap\beta cl(A\cap B)^c$

$\subseteq(\beta cl(A)\cap\beta cl(B))\cap(\beta cl(A^c)\cup\beta cl(B^c))$

$=[\beta cl(A)\cap(\beta cl(B))\cap\beta cl(A^c)]\cup[(\beta cl(A)\cap$
$(\beta cl(B))\cap\beta cl(B^c))$

$=(\beta IBd(A)\cap(\beta cl(B))\cup(\beta IBd(B)\cap(\beta cl(A))$

$\Longrightarrow I\beta Bd(A\cap B)\subseteq(I\beta Bd(A)\cap\beta cl(B))\cup(I\beta Bd(B)\cap\beta cl(A)).$

Example 3.23.

Let (X,τ) be an IFTS defined in example 3.2.

and $C=\langle x,\left(\frac{a}{0.2},\frac{b}{0.4}\right),\left(\frac{a}{0.8},\frac{b}{0.4}\right)\rangle$, Then $I\beta Bd(C\cap B)=C$, and

$(I\beta Bd(C)\cap\beta cl(B))\cup(I\beta Bd(B)\cap\beta cl(C))=\langle x,\left(\frac{a}{0.3},\frac{b}{0.4}\right),\left(\frac{a}{0.7},\frac{b}{0.4}\right)\rangle.$

Then $I\beta Bd(A\cap B)\subseteq(I\beta Bd(A)\cap\beta cl(B))\cup(I\beta Bd(B)\cap\beta cl(A))$

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