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The Adomian decomposition method for solving a system of non-linear equations.

A Thesis submitted to the department of mathematics in partial fulfillment of the requirements for the degree of master of science in mathematics

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بسم الله الرحمن الرحيم

﴿قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ﴾[1]

صدق الله العظيم

(1) سورة البقرة الآية (31)

DEDICATION

I would like to dedicate this thesis to who gave me a favor after ALLAH almighty, and by their encouragement I continued, to my parents, may ALLAH give them prolong age.

To whom the words of thanks are stand unable to give him his right

My husband

My brother and sisters

To every researcher in the paths of science and knowledge.

ACKNOWLEDGMENT

All gratitude and thanks are due to almighty ALLAH who inspired me to bring forth to light this dissertation. I would like to express my thanks and respectful to my supervisor Dr. Ali Aoun, for all his valuable guidance, constructive suggestions and constant encouragement during the preparation of the thesis.

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Abstract

This thesis discusses one of the unified methods used to solve linear, non-linear equations, ordinary and partial differential equations known as the Adomian decomposition method. The Adomian decomposition method is discussed in details to include its background information, polynomials, convergence, formulation, boundary conditions, applications and advantages. Once the Adomian decomposition method has been thoroughly presented, its applications in systems of non-linear equations will be discussed in details and numerical examples will be provided to show the effectiveness of this method. Results of these numerical examples show that the Adomian decomposition method is an effective method to solve systems of non-linear equations with ease and favorable accuracy.

Introduction

Developing an unified method to solve linear, non-linear equations, ordinary and partial differential equations is one of many goals mathematicians aim to achieve. This goal has always been difficult to achieve due to the irregularities associated with non-linear equations, and this difficulty is increased when dealing with system of such equations. The goal of this thesis is to give a detailed analysis of this method and its applications in systems of non-linear equations.

One of researchers who succeeded in developing an unified theory to solve linear and nonlinear ordinary and partial differential equations was the mathematician known as George Adomian. George Adomian developed his theory during the period of 1970-1990, and named after himself as the “Adomian decomposition method“ [8]. This method is a semi-analytical method used to solve ordinary and partial linear and nonlinear differential equations. This method employs polynomials known as “Adomian polynomials“ that allow for solution convergence of the nonlinear portion of the equation without the need to linearize the equation. This is considered a crucial aspect of the Adomian decomposition method. This method enjoys greater flexibility than the direct Taylor series expansion since the polynomials generalize mathematically to a Maclaurin series about an arbitrary external parameter [13]. The Adomian decomposition method, henceforth referred to as ADM, has received extensive amount of research due to its ability to be applied to real world problems in both the science and engineering disciplines. Some of these studies aim to study the methods ability to solve nonlinearities including product, polynomial, exponential, hyperbolic, … etc. [4] [5] [6] [7], while other studies aimed to modify and enhance this method to improve convergence.

Numerical examples will be provided and solved to show case the effectiveness of this method.

This thesis will be in follow the following :

Chapter one will include a brief summary of the background information needed to understand and construct the rest of the thesis.

Chapter two will provide detailed information on the Adomian decomposition method.

Chapter three will discuss the main topic of this thesis, which is the solving of non-linear system of equations. This chapter will also solve two numerical examples, to show the effectiveness of the Adomian decomposition method, and will include the conclusions made in this thesis .

Chapter One

Fundamental concepts

1.1 Basic definitions

1.2 Solution of first order ODE

1.3 System of equations

1.4 System of differential equations

1.5 System of differential equations solution method

1.6 System of non-linear equations

Fundamental concepts

In this chapter we give a brief introduction and a background information summary of differential equations, moreover system of equations and system of differential equations will be provided to understand the basics of this thesis, are taken from [3][12][16][18][22][25].

1.1 Basic definitions

Definition 1.1

A differential equation of n^{th} order is considered linear if it takes on the following form:

$$a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \cdots + a_1(x)y'(x) + a_0(x)y(x) = g(x) \tag{1.1}$$

where g is any given function of the independent variable x and it is assumed that $a_n(x) \neq 0$. On the other hand, any equation that does not take on the form of equation (1.1) is considered a non-linear differential equation of n^{th} order. For linear differential equations of 1st order, the previous equation then becomes:

$$y' + p(x)y = q(x) \tag{1.2}$$

The most important aspect of linear differential equations, is that there are no products of the function $y(x)$ not are there for its derivatives and neither of them occur to any power other than the first. The linearity of the equation is determined solely on $y(x)$ and its derivatives and the coefficients $a_0(x), \dots, a_n(x)$ and $g(x)$ can be zero or non-zero functions, constants or non-constant functions, linear or non-linear functions without affecting the linearity of the equation. A linear differential equation can be ordinary or partial depending on the form of the derivatives.

Example of linear ordinary differential equation include:

$$ay'' + by' + cy = g(x)$$

$$y^{(4)} + 10y''' - 4y' + 2y = \cos(t)$$

The following equation represents a non-linear equation:

$$\sin(y)\frac{\partial^2 y}{\partial x^2} = (1-y)\frac{\partial y}{\partial x} + y^2 e^{-5y}$$

Definition 1.2

Differential equations are considered homogeneous if they satisfy either of the two following conditions:

1. If a first order differential equation can be written in the following form, then it is considered homogeneous:

$$f(x,y)dy = g(x,y)dx \tag{1.3}$$

where f and g are homogeneous functions of the same degree of x and y. In this case, the change of variable $y = ux$ leads to an equation of the form:

$$\frac{dx}{d} = h(u)du \tag{1.4}$$

which can be solved with ease via integration of both members.

2. A differential equation is homogenous if it is a homogenous function of the unknown function and its derivatives. For linear differential equations, this translates to the equation having no constant terms. Therefore, the solutions of any linear ordinary differential equation of any order may be deduced by integration from the solution of the homogeneous equation obtained by removing the constant term.

Any differential equation does not satisfy either of these conditions are considered non-homogenous differential equations. For example, take the following simple homogenous differential equation:

$$y'' = xy$$

The equation is considered homogenous since only the unknown function y and its derivatives are present. Now take the following non-homogeneous differential equation:

$$y'' = xy + (x + 1)$$

This equation is considered non-homogeneous since it contains the term $(x + 1)$, which does not involve the unknown function y and its derivatives.

A differential equation is also considered homogenous if both x and y have the same power. for example,

$$(-x + y)\frac{dy}{dx} = 2y$$

Since both $(-x + y)$ and $2y$ both have the same power (1), they are considered homogeneous. As for linear ordinary differential equation of order n, it is considered homogenous if the following form is taken:

$$a_n(x)y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \cdots + a_1(x)y' + a_0(x)y = 0 \qquad (1.5)$$

In other words, if all the terms are proportional to a derivative of y (or y itself) and there is no term that contains a function of x alone then the linear ordinary differential equation is considered homogenous. It should be mentioned that the existence of a constant term is a sufficient condition for an equation to be non-homogeneous.

Definition 1.3

Let $D \subset \mathbb{R}^n$, $f(x,y)$ continous function, we say that $f(x,y)$ satisfies Lipshitz condition if there exist constant $L > 0$, such that $\frac{\partial f}{\partial y}$ exists and its bounded in D, $\forall\ x, y \in D$.

Definition 1.4

The solution of ODE is a function that satisfies the differential equation and the derivatives exist.

1.2 Solution of first order ODE

There are many ways to solve a first order differential equation, examples of these include:

1.2.1 Separable differential equations

A separable first order differential equation is any differential equation that can be written in the following form:

$$\frac{dy}{dx} = f(x,y) = \frac{A(x)}{B(y)} \tag{1.6}$$

This is solved as follows:

$$B(y)dy = A(x)dx \Longrightarrow \int B(y)dy = \int A(x)dx \tag{1.7}$$

This provides an implicitly defined solution of $y(x)$.

1.2.2 Homogenous differential equations

A homogenous differential equation of the first order can be written in the form:

$$y' = f\left(\frac{y}{x}\right), \tag{1.8}$$

and can be made separable under the change of variable:

$$y = ux, \tag{1.9}$$

and then:

$$dy = udx + xdu \tag{1.10}$$

1.2.3 Exact differential equations

Consider the following standard form of ordinary differential equations:

$$M(x, y)dx + N(x, y)dy = 0 \tag{1.11}$$

The necessary and sufficient condition for the D.f to be exact is:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{1.12}$$

This means the function $f(x, y)$ exists such that:

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = M(x, y)dx + N(x, y)dy = 0 \tag{1.13}$$

Then we have two equations:

$$\frac{\partial f}{\partial x} = M(x, y) \text{ and } \quad \frac{\partial f}{\partial y} = N(x, y) \tag{1.14}$$

Starting with the first equation and integrating both sides with respect to x:

$$\int \frac{\partial f}{\partial x}dx = \int M(x, y)dx \tag{1.15}$$

which becomes:

$$f(x, y) = \int M(x, y)dx + h(y) \tag{1.16}$$

where h is an arbitrary function of y. Now to use the second equation, take the partial derivative of f with respect to y and set it equal to $N(x,y)$:

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\int M(x,y)dx + h'(y) = N(x,y) \tag{1.17}$$

Once we solve $h'(y)$, we integrate it to find $h(y)$. Then, we have found our solution:

$$\int M(x,y)dx + h(y) = c \tag{1.18}$$

1.2.4 Integrating factor

If the following equation is not exact:

$$M(x,y)dx + N(x,y)dy = 0$$

and $\mu(x,y)$ is the integrating factor for this equation, then:

$$\mu M dx + \mu N dy = 0 \tag{1.19}$$

will be an exact equation i.e.

$$\frac{\partial(\mu M)}{dy} = \frac{\partial(\mu N)}{\partial x} \tag{1.20}$$

$$\mu\frac{\partial M}{\partial y} + M\frac{\partial \mu}{\partial y} = \mu\frac{\partial N}{\partial y} + N\frac{\partial \mu}{\partial y} \tag{1.21}$$

and

$$\mu\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = N\frac{\partial \mu}{\partial x} - M\frac{\partial \mu}{\partial y} \tag{1.22}$$

Now, we have two cases to find the integrating factor:

First case: If $\mu = \mu(x)$, that means $\frac{\partial \mu}{\partial x} = \frac{d\mu}{dx}$, $\frac{\partial \mu}{\partial y} = 0$ and we can write equation (1.22) in the following form:

$$\frac{1}{\mu}\frac{d\mu}{dx} = \frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right] = f(x) \tag{1.23}$$

$$\frac{d\mu}{\mu} = f(x)dx \Longrightarrow \ln\mu = \int f(x)dx \tag{1.24}$$

Then, the integrating factor is:

$$\mu = e^{\int f(x)dx} \tag{1.25}$$

Second case: If $\mu = \mu(y)$, that means $\frac{\partial\mu}{\partial y} = \frac{du}{dy}, \frac{\partial\mu}{\partial x} = 0$ and by the same way we get:

$$\frac{1}{\mu}\frac{du}{dy} = \frac{1}{M}\left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] = g(y) \tag{1.26}$$

Then,

$$\mu = e^{\int g(y)dy} \tag{1.27}$$

1.2.5 Linear ODE of 1st order

Consider the ODE of the form:

$$y' + P(x)y = Q(x) \tag{1.28}$$

where P and Q are given functions of x, defined on a certain interval I. This equation is called linear equation.

To solve this equation, we have to change it to exact equation. Now put equation (1.28) to the form:

$$dy + P(x)ydx = Q(x)dx \Longrightarrow dy + (Py - Q)dx = 0 \tag{1.29}$$

Suppose there exist $\mu = \mu(x)$ an integral factor that make the previous equation an exact equation. Thus:

$$\mu dy + \mu\big(P(x)y - Q(x)\big)dx = 0 \tag{1.30}$$

Let $M = \mu\big(P(x)y - Q(x)\big) \Longrightarrow \frac{\partial M}{\partial y} = \mu P(x)$ (1.31)

and

$$N = \mu \Longrightarrow \frac{\partial N}{\partial x} = \frac{d\mu}{dx} \tag{1.32}$$

As we know:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Longrightarrow \mu P = \frac{d\mu}{dx} \tag{1.33}$$

We can use separable equations for last equation to get:

$$\frac{d\mu}{\mu} = P(x)dx \tag{1.34}$$

and integrating both sides, we get:

$$\ln \mu = \int P(x)dx \tag{1.35}$$

i.e. the integral equation is:

$$\mu = e^{\int P(x)dx} \tag{1.36}$$

By multiplying equation (1.28) by μ, the equation becomes:

$$\mu y' + \mu P(x)y = \mu Q(x) \tag{1.37}$$

Thus,

$$\frac{d}{dx}(\mu y) = \mu Q(x) \tag{1.38}$$

By integrating:

$$\mu y = \int \mu Q(x)dx + c \tag{1.39}$$

i.e. the general solution is:

$$y = \mu^{-1}[\int \mu Q(x)dx + c] \tag{1.40}$$

1.2.6 Bernoulli's equation

A differential equation of the form:

$$y' + P(x)y = Q(x)y^n \tag{1.41}$$

where n is a real parameter such that n does not equal 0 or 1 is said to be of Bernoulli type. Letting:

$$z = y^{1-n}, \quad z' = (1-n)y^{1-n}y' \tag{1.42}$$

The equation transforms into:

$$z' + (1-n)P(x)z = (1-n)Q(x) \tag{1.43}$$

and thus it becomes a linear equation and is solved as a linear equation.

1.3 System of equations

A system of equations is a finite set of equations for which common solutions are sought. An equation system is usually classified in the same manner as single equations. However, this study will focus on two main classifications which are the linear and non-linear equations. The general form of a system of m linear equations with n unknowns is written as:

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{gathered} \tag{1.44}$$

where x_1, x_2, ..., x_n are the unknowns and a_{11}, a_{12}, ..., a_{mn} are the systems coefficients and b_1, b_2, ..., b_m are the constant terms. The coefficients and unknowns are often real or complex numbers, however, integers, rational numbers, polynomials and elements can also be observed in systems of linear equations. Generally, the relationship between the number of equations and the number of unknowns is what determines the behavior of a linear system. The following points summarize the possible general behaviors:

1. A system with fewer equations than unknowns has infinitely many solutions, but it may have no solution. Such a system is known as an underdetermined system.
2. A system with the same number of equations and unknowns has a single unique solution.
3. A system with more equations than unknowns has no solution. Such a system is also known as an overdetermined system.

The term “generally” is used since not all system of equations may follow the general behavior and may behave differently for specific values of the coefficients of the equations. A system of linear equations behaves differently from the general case if the equations are linearly dependent, or if it is inconsistent and has no more equations than unknowns. A system of linear equations can be categorized further depending on its properties into independent and consistent system of linear equations. The equations of a linear system are said to be independent if none of the equations can be derived algebraically from the others. When the equations are independent, each equation contains new information about the variables, and removing any of the equations increases the size of the solution set. Alternatively, the equations of a linear equation are not considered independent if any of the equations can be derived from the other. This can be via multiple algebraic methods such as multiplication, addition and subtraction. This can include all or part of the systems equation and since the equations can be derived from one another, any one of the equations can be removed without affecting the solution set. As for consistency, a linear system is inconsistent if it has no solution, and otherwise it is said to be consistent. When the system is inconsistent, it is possible to derive a contradiction from the equations, that may always be rewritten as the statement 0 = 1. It is possible for three linear equations to be inconsistent, even though any two of them are consistent together. In general, inconsistencies occur if the left-hand sides of the equations in a system are linearly dependent, and the constant terms do not satisfy the dependence relation. A system of equations whose left-hand sides are linearly independent is always consistent. As for whether the system of linear equations is considered homogenous or not, it is considered homogenous if all of the constant terms are zero, i.e. follows this general form:

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{gathered} \tag{1.45}$$

A homogeneous system is equivalent to a matrix equation of the form

A x =0. Any system of equations that don't satisfy these conditions are considered non-homogenous.

A nonlinear system is a system in which the change of the output is not proportional to the change of the input, also non-linear system of equations is a system in which at least one of the variables has an exponent other than 1 and/or there is a product of variables in one of the equations. Most systems are nonlinear in nature, which is why it is of great interest to many science disciplines such as engineering, mathematics … etc. Nonlinear dynamical systems, describing changes in variables over time, may appear chaotic, unpredictable, or counterintuitive, contrasting with much simpler linear systems. A nonlinear system is described in mathematics using nonlinear system of equations, which are equations that do not fall under the category of linear equations. Systems can be defined as nonlinear, regardless of whether known linear functions appear in the equations. As nonlinear dynamical equations are difficult to solve, nonlinear systems are commonly approximated by linear equations (linearization).

1.4 System of differential equations

In most real life problems, a system is governed by more than one differential equation especially when two variables effect one another, this is known as a system of differential equation. There are numerous applications of systems of differential equations, perhaps the most well-known example is the predator-prey interactions. Since the number of preys effect the number of predators in a system and the number of predators affects the number of prey. Since both of these variables affect one another, their interaction is modeled using a system of differential equations. To further explain this, the following equation represents a first order linear differential equation:

$$x'_1 = 2x_1 - 4x_2$$
$$x'_2 = 3x_1 + x_2$$

Since the value of x_2 is dependent on x_1, and the value of x_1 is dependent on x_2, this system is known as a coupled system.

A system of linear differential equations is a set of linear equations relating a group of functions to their derivatives. Since these equations include the function and its derivatives, each of these linear equations is differential equation in of itself. For example:

$$f'(x) = f(x) + g(x)$$

This equation relates f' to f and g. Therefore, it is linear while the equation $f' = fg$ is not linear because the term fg isn't linear. To summarize this, the linearity of the system of differential equation depends on the linearity of its equations. Systems of differential equations can be used to model a variety of physical systems but linear systems are the only systems that can be consistently solved explicitly.

1.5 System of differential equations solution method

Solving a system of differential equations usually entail converting it to matrix form first before solving it. To help show this method of convergence, take the following example:

$$\begin{aligned} x_1' &= 4x_1 + 7x_2 \\ x_2' &= -2x_1 - 5x_2 \end{aligned}$$

To convert this system into a matrix, the first step is to write this system in way that each side becomes a vector, as shown in the following expression:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 4x_1 + 7x_2 \\ -2x_1 - 5x_2 \end{pmatrix}$$

The next step is to rewrite the right side as a matrix multiplication:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 4 & 7 \\ -2 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

By defining $\vec{x}$ as:

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

and $\vec{x}'$ as:

$$\vec{x}' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix}$$

The system takes the following matrix form:

$$\vec{x}' = \begin{pmatrix} 4 & 7 \\ -2 & -5 \end{pmatrix} \vec{x}$$

The general form of the matrix takes the following form:

$$\vec{x}' = A\vec{x} + \vec{g}(t) \tag{1.46}$$

where A is an $n \times n$ matrix and $\vec{x}$ is a vector whose components are the unknown functions in the system. The system is considered homogeneous should $\vec{g}(t) = 0$. Otherwise, it is considered non-homogenous. Now, the method of converting a system to a matrix is covered, the next step is to show how to solve the equation; to show this, consider the following homogeneous system of differential equations written in matrix form:

$$\vec{x}' = A\vec{x} \tag{1.47}$$

By starting at $n = 1$, the system is reduced to a simple linear or separable first order differential equation:

$$x' = ax \tag{1.48}$$

Which has the following solution:

$$x(t) = ce^{at} \tag{1.49}$$

By using this solution as a guide, we will attempt to develop a solution for a general n to see if the following equation will be the solution:

$$\vec{x}(t) = \vec{\eta}e^{rt} \tag{1.50}$$

It should be noted that the only real difference is the constant in front of the exponential is set as a vector. All that is left to do is to "plug" this into the differential equation and observe its result. First notice that the derivative is given as:

$$\vec{x}'(t) = r\vec{\eta}e^{rt} \tag{1.51}$$

By plugging the previous guess into the differential equation it becomes:

$$r\vec{\eta}e^{rt} = A\vec{\eta}e^{rt}$$

$$(A\vec{\eta} - r\vec{\eta})e^{rt} = \vec{0}$$

$$(A - rI)\vec{\eta}e^{rt} = \vec{0} \tag{1.52}$$

Since the exponentials are not zero and by dropping that portion, it is clear that for equation (1.50) to be a solution of equation (1.47), the following equation must be true:

$$(A - rI)\vec{\eta} = 0 \tag{1.53}$$

or that r and $\vec{\eta}$ must be eigenvalue and eigenvector for the matrix A. Therefore, to solve equation (1.47), the eigenvalue and eigenvector of the matrix A must be found and then a solution can be formed using equation (1.50). There are three possible cases that the eigenvalue can be, which are real, distinct eigenvalues, complex eigenvalues and repeated eigenvalues.

However, these on their own do not show how to solve a system of differential equation, the following facts are needed to do this:

1. If $\vec{x}_1(t)$ and $\vec{x}_2(t)$ are two solutions to a homogeneous system, then: $c_1\vec{x}_1(t) + c_2\vec{x}_2(t)$ is also considered a solution to the system.
2. Suppose that A is an $n \times n$ matrix and $\vec{x}_1(t), \vec{x}_2(t), \dots, \vec{x}_n(t)$ are solutions to a homogeneous system. In other words, X is a matrix with an i^{th} solution.

   Now define , $W = \det(X)$, W is called the Wronskian. If $W \neq 0$, then the solution form a fundamental set of solutions and the general solution to the system is:

$$\vec{x}(t) = c_1\vec{x}_1(t) + c_2\vec{x}_2(t) + \cdots + c_n\vec{x}_n(t) \tag{1.54}$$

It should be noted that if a fundamental set of solutions is obtained, the solutions are also going to be linearly independent. Similarly, if we have a set of linearly independent solutions, then they will also be a fundamental set of solutions since the Wronskian will not be zero.

1.6 System of non-linear differential equations

Equilibrium is a state of a system which does not change.

Definition 1.5

An equilibrium point (fixed point) is a steady state, that is a rest state, of system. When a system is found at an equilibrium point at some time t_0 then it will remain in it for $t > t_0$.

A system of differential equations is considered non-linear if it cannot be written in the form of $x_0 = Ax$ for a matrix A. systems of non-linear differential equations cannot be solved using the methods of linear differential equation systems, new methods are needed to discern the nature of the equilibria in non-linear systems in order to linearize the system. For an example take the following equations:

$$x' = x + \binom{\alpha}{\beta} \tag{1.55}$$

This system has an equilibrium point $\binom{-\alpha}{-\beta}$. By introducing a change in coordinates $\bar{x}_1 = x_1 + \alpha, \bar{x}_2 = x_2 + \beta$ so that the system becomes:

$$\bar{x}' = \bar{x} + \binom{\alpha}{\beta} = x - \binom{\alpha}{\beta} + \binom{\alpha}{\beta} = x \tag{1.56}$$

This new system $\bar{x}' = \bar{x}$ is linear with a unique equilibrium point at (0,0). Moreover, $\bar{x}' = x'$ for any $x \in \mathbb{R}^2$, since $\bar{x}' = \left(x - \binom{\alpha}{\beta}\right)' = x'$.

Proposition 1.1

For any system of n-dimensional differential equations of the form $x' = Ax + V$ for some $V \in \mathbb{R}^n$ and with unique equilibrium point x_e, then the change of coordinates $\bar{x} = x - x$ yields a linear system of differential equations $\bar{x}' = A\bar{x}$ with unique equilibrium point 0.

Proof:

Since x_e is an equilibrium point of the system, $Ax_e + V = 0$.

Then, $\bar{x}' = A(\bar{x} + x_e) + V = A\bar{x} + (Ax_e + V) = A\bar{x}$.

Then, we have from the previous example that $\bar{x}' = x'$ for any $x \in \mathbb{R}^2$, as our proof in $\mathbb{R}^2$ did not require any specific characteristics of $\mathbb{R}^2$.

Therefore, $0 = x_e' = (xe - xe)' = \bar{0}, i.e.$ $\bar{x} = A\bar{x}$ *has equilibrium point 0. Since* x_e *is a unique equilibrium point, 0 is the only element of* $\mathbb{R}^n$ *with* $\bar{x} = 0$.

More generally, we have the notion of conjugacy.

Definition 1.6

Let $F : X \to X$, G : Y→Y, F and G are topologically conjygate if there exists a homeomorphism h : X→Y such that $h^0 F = G^0 h$.

That is, two systems are conjugate if there exists a "change of coordinates" from one system to the other.

In studying nonlinear systems, we are notably interested in systems which are conjugate to linear systems. However, most nonlinear systems are not conjugate to linear equations. Most nonlinear systems, in fact, cannot be solved to arrive at a general equation. For example, consider the following system:

$$x' = x - 3y + x^3$$
$$y' = -x + y - 2y^4$$

This system cannot be solved explicitly, but the nature of the equilibrium point (0,0) can be discerned. Since it is known that x, y, x^3 and $2y^4$ tend to 0 much faster than the linear terms. Therefore, sufficiently close to (0,0), the system behaves similarly to:

$$x' = x - 3y$$
$$y' = -x + y$$

The linear system has eigenvalues $\lambda = 1 \pm i\sqrt{3}$, meaning the equilibrium point is a spiral source. Therefore, in the nonlinear system, we know at least that the equilibrium point is a source.

Chapter Two

Adomian decomposition method

2.1 Adomian decomposition method principle

2.2 Adomian polynomials

2.3 Convergence analysis of the (ADM) method

2.4 Convergence order of (ADM) method

2.5 Formulation of Adomian polynomials for nonlinear cases

2.6 Boundary conditions

2.7 Applications of the (ADM) method

2.8 Advantages and Disadvantages of the (ADM) method

2.1 Adomian decomposition method principle

In order to show the ADM's principle, consider the following nonlinear ordinary differential equation (ODE):

$$Lu + Nu + Ru = g \tag{2.1}$$

where u is the unknown function, L is the linear differential operator of high order which is easily invertible, N is the nonlinear operator, R is the remaining linear part, g is the given function (source).

Multiply the inverse of the linear differential operator L which is (L^{-1}) of both sides of the equation, which results in the following expression:

$$L^{-1}(Lu + Nu + Ru = g) \tag{2.2}$$

It should be mentioned that the choice of L and consequently L^{-1} are determined by the particular equation that is to be solved i.e. the choice of L and L^{-1} is non-unique. By multiplying L^{-1} into the brackets we gain the following equation:

$$L^{-1}Lu = L^{-1}g - L^{-1}N(u) - L^{-1}R(u) \tag{2.3}$$

This yields the following equation:

$$u - \phi = L^{-1}g - L^{-1}N(u) - L^{-1}R(u) \tag{2.4}$$

where ϕ is presented from the initial conditions or from the boundary conditions or both, it depends on how we choose differential operator that solve the given problem.

In the (ADM) method, it is assumed that the solution (u) of the functional equation can be decomposed into infinite series as follows:

$$u = \sum_{n=0}^{\infty} u_n \tag{2.5}$$

The (ADM) method also assumes that the nonlinear term $N(u)$ can be written as an infinite series as follows:

$$N(u) = \sum_{n=0}^{\infty} A_n \tag{2.6}$$

where the A_n's are the Adomian polynomials, which as mentioned before is the cornerstone of the Adomian method. The method of obtaining the Adomian polynomials will be explained in the following section. By substituting both series into equation (2.4), we get,

$$\sum_{n=0}^{\infty} u_n = \phi + L^{-1}g - L^{-1}\sum_{n=0}^{\infty} A_n - L^{-1}\sum_{n=0}^{\infty} R(u_n) \tag{2.7}$$

From the previous equation the following algorithm can be obtained:

$$u_0 = \phi + L^{-1}g, \quad u_{n+1} = -L^{-1}(A_n + Ru_n), \qquad n = 0, 1, 2, \ldots \tag{2.8}$$

By obtaining u_0, the other terms of u can be determined respectively. It should be mentioned that if any values of u_n equals to zero then all the terms come after are zero as well.

2.2 Adomian polynomials

As mentioned before, the Adomian decomposition method relies heavily on Adomian polynomials (A_n). These polynomials are determined via a general formula that was given by George Adomian in 1992, this formula is expressed as:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0}, \qquad n = 0,1,2,3 \ldots \tag{2.9}$$

Using formula (2.9), the three first terms of the Adomian polynomials can be determined and expressed as follows:

$$A_0 = \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[N\left(\sum_{i=0}^{0}\lambda^i u_i\right)\right]_{\lambda=0} = N(u_0) \tag{2.10}$$

$$A_1 = \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[N\left(\sum_{i=0}^{1}\lambda^i u_i\right)\right]_{\lambda=0} = \frac{d}{d\lambda}[N(\lambda^0 u_0 + \lambda^1 u_1)]_{\lambda=0} = [N'(\lambda^0 u_0 + \lambda^1 u_1)]_{\lambda=0}(u_1) = u_1 N'(u_0) \tag{2.11}$$

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N\left(\sum_{i=0}^{2}\lambda^i u_i\right)\right]_{\lambda=0} = \frac{1}{2!}\frac{d^2}{d\lambda^2}[N(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2)]_{\lambda=0} = \frac{1}{2!}\frac{d}{d\lambda}[N'(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2)(u_1 + 2\lambda u_2)]_{\lambda=0} = \frac{1}{2!}[N'(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2)(2u_2) + N''(\lambda^0 u_0 + \lambda^1 u_1 + \lambda^2 u_2)(u_1 + 2\lambda u_2)^2]_{\lambda=0} = \frac{u_1^2}{2!}N''(u_0) + u_2 N'(u_0) \tag{2.12}$$

In the (ADM) method, the nonlinear function $N(u)$ found around initial function u_0, can be obtained in a similar fashion to the Traylor series expansion, as shown below:

$$N(u) = N(u_0) + N'(u_0)(u - u_0) + \frac{1}{2!}N''(u_0)(u - u_0)^2 + \cdots \tag{2.13}$$

In the ADM method, u can be expressed as:

$$u = \sum_{n=0}^{\infty} u_n = u_0 + u_1 + u_2 + \cdots \tag{2.14}$$

By substituting equation (2.14) into expansion (2.13), the following expression is obtained:

$$N(u) = N(u_0) + N'(u_0)(u_1 + u_2 + \cdots) + \frac{1}{2!}N''(u_0)(u_1 + u_2 + \cdots)^2 + \frac{1}{3!}N'''(u_0)(u_1 + u_2 + \cdots)^3 + \cdots \tag{2.15}$$

By taking apart the expansion terms, the previous expansion becomes:

$$N(u) = N(u_0) + N'(u_0)(u_1) + N'(u_0)(u_2) + N'(u_0)(u_3) + \cdots + \frac{1}{2!}N''(u_0)(u_1)^2 + \frac{1}{2!}N''(u_o)u_1u_2 + \frac{1}{2!}N''(u_0)u_2u_1 + \frac{1}{2!}N''(u_0)(u_2)^2 + \frac{1}{2!}N''(u_0)u_1u_3 + \frac{1}{2!}N''(u_0)u_3u_1 + \cdots + \frac{1}{3!}N''''(u_0)(u_1)^3 + \frac{1}{3!}N'''(u_0)(u_1)^2u_2 + \frac{1}{3!}N'''(u_0)u_2(u_1)^2 + \frac{1}{3!}N'''(u_0)u_1u_2u_1 + \cdots \tag{2.16}$$

By recording the terms and determining the order of each term which depend on both the subscript and the exponent of the u_n's. This results into the following expression:

$$N(u) = N(u_0) + N'(u_0)u_1 + N'(u_0)u_2 + \frac{1}{2!}N''(u_0)u_1^2 + N'(u_0)u_3 + \frac{2}{2!}N''(u_0)u_1u_2 + \frac{1}{3!}N'''(u_0)u_1^3 + N'(u_0)u_4 + \frac{1}{2!}N''(u_0)u_2^2 + \frac{2}{2!}N''(u_0)u_1u_3 + \frac{3}{3!}N'''(u_0)u_1^2u_2 + \cdots \tag{2.17}$$

By comparing the terms from equation (2.17) with the terms of the assumption made in equation (2.6), the values of A_n's can be written as follows:

$$A_0 = N(u_0)$$

$$A_1 = u_1N'(u_0)$$

$$A_2 = u_2N'(u_0) + \frac{u_1^2}{2!}N''(u_0)$$

$$A_3 = u_3N'(u_0) + \frac{2u_1u_2}{2!}N''(u_0) + \frac{u_1^3}{3!}N'''(u_0)$$

$$\cdots$$

It is clear that, these are the same values gained from the Adomian general formula that is used to determine the Adomian polynomials, as shown in equation (2.9).

2.3 Convergence analysis of the (ADM) method

The proof of convergence for the Adomian method was first provided by Cherruault [13] [14], where he used fixed point theorems for abstract functional equations. To show the convergence of the (ADM) method consider this general functional equation:

$$u - N(u) = f, \qquad \text{for } u \in H \tag{2.18}$$

where H is the Hilbert space and $N: H \to H$ and f is any given function in H. As aforementioned, the Adomian decomposition method assumes a series solution for u given by equation (2.5), while the nonlinear term $N(u)$ is given as the sum of series as shown in equation (2.6) and the Adomian polynomials are given by equation (2.9). By substituting equations (2.5) and (2.6) into equation (2.18) the following expression is obtained:

$$\sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} A_n = f \tag{2.19}$$

The recursive terms are obtained from the following algorithm:

$$\begin{gathered} u_0 = f \\ u_{n+1} = A_n(u_0, u_1, \dots, u_n) \end{gathered} \tag{2.20}$$

The Adomian decomposition method uses an iterative scheme which is equivalent to finding the sequence $S_n = u_1 + u_2 + u_3 + \cdots + u_n$, which is defined by:

$$\begin{gathered} S_0 = 0, \\ S_{n+1} = N_n(u_0 + s_n), \end{gathered} \tag{2.21}$$

where

$$N_n(u_0 + s_n) = \sum_{n=0}^{i} A_i \qquad (2.22)$$

If the following limits exist in a Hilbert space (H):

$$S = \lim_{n\to\infty} S_n\,, \quad N = \lim_{n\to\infty} N_n \qquad (2.23)$$

Then, S solves the functional equation $S = N(u_0 + S)$ in H.

The following theorem shows proof of the ADM method's convergence:

Theorem 2.1

Let N be a nonlinear operator from a Hilbert space H where: $N: H \to H$ and u be the exact solution of equation (2.18). The decomposition series $\sum_{n=0}^{\infty} u_n$ of u convergence to u when:

$$\exists \alpha < 1, \|u_{n+1}\| \le \alpha \|u_n\|, \forall n \in \mathbb{N} \cup \{0\}$$

Proof

We have the sequence

$$S_n = u_1 + u_2 + u_3 + \cdots + u_n \qquad (2.24)$$

It is necessary to show that the previous sequence is a Cauchy sequence in the Hilbert space (H). To achieve this let:

$$\|S_{n+1} - S_n\| = \|u_{n+1}\| \le \alpha \|u_n\| \le \alpha^2 \|u_{n-1}\| \le \cdots \le \alpha^{n+1} \|u_0\| \qquad (2.25)$$

Since

$$\begin{aligned}\|S_m - S_n\| &= \|(S_m - S_{m-1}) + (S_{m-1} - S_{m-2}) + \cdots + (S_{n+1} - S_n)\| \\ &\le \|S_m - S_{m-1}\| + \|S_{m-1} - S_{m-2}\| + \cdots + \|S_{n+1} + S_n\| \\ &\le \alpha^m \|u_0\| + \alpha^{m-1} \|u_0\| + \cdots + \alpha^{n+1} \|u_0\|\end{aligned}$$

$$= (\alpha^{m} + \alpha^{m-1} + \cdots + \alpha^{n+1}\|u_0\|)$$

$$\leq (\alpha^{n+1} + \alpha^{n+2} + \cdots)\|u_0\|$$

Then

$$\|S_m - S_n\| = \frac{\alpha^{n+1}}{1-\alpha}\|u_0\|, \quad for\ n, m \in \mathbb{N}, m \geq n \qquad (2.26)$$

Thus S_m converges to S_n and

$$\lim_{n,m\to\infty} \|S_m - S_n\| = 0 \qquad (2.27)$$

From the equation (2.27), the sequence $\{S_n\}_{n=0}^{\infty}$ is a Cauchy sequence in the Hilbert space (H). Hence,

$$\lim_{n\to\infty} S_n = S \qquad \text{for } S \in H, \qquad (2.28)$$

where

$$S = \sum_{n=0}^{\infty} u_n$$

Solving equation (2.18) is the same as solving the functional $N(u_0 + S)$; by assuming that N is a continouse operator, we get:

$$N(u_0 + S) = N\left(\lim_{n\to\infty}(u_0 + S_n)\right) = \lim_{n\to\infty} N(u_0 + S_n)$$

$$= \lim_{n\to\infty} S_{n+1} = S \qquad (2.29)$$

Therefore, the solution of equation (2.18) is S.

2.4 Convergence order of (ADM) method

The convergence order of the (ADM) method was defined by Babolian and Biazer [11] as:

Definition 2.2

Let S_n be a sequence that converges to S. If there exist two constants $p \in \mathbb{N}$, $c \in \mathbb{R}$, such that:

$$\lim_{n\to\infty} \left|\frac{S_{n+1}-S}{(S_n-S)^p}\right| = c \qquad (2.30)$$

Then the order of convergence of S_n is p.

To determine the order of convergence of S_n, the Taylor expansion of $N(S_n + u_0)$ around the point $(S + u_0)$ can be considered:

$$N(S_n+u_0) = N(S+u_0) + N'(S+u_0)(S_n-S) + \frac{1}{2!}N''(S+u_0)\,(S_n-S)^2 + \cdots$$

$$+\frac{1}{m!}N^{(m)}(S+u_0)(S_n-S)^m + \cdots$$

$$N(S_n+u_0) - N(S+u_0) = N'(S+u_0)(S_n-S) + \frac{1}{2!}N''(S+u_0)(S_n-S)^2 + \cdots$$

$$+\frac{1}{m!}N^{(m)}(S+u_0)(S_n-S)^m + \cdots \qquad (2.31)$$

Since $N(S+u_0) = S$ and $N(S_n+u_0) = S_{n+1}$, Therefore, the previous equation becomes:

$$S_{n+1} - S = N'(S+u_0)(S_n-S) + \frac{1}{2!}N''(S+u_0)(S_n-S)^2 + \cdots + \frac{1}{m!}N^{(m)}(S+u_0)(S_n-S)^m + \cdots$$
(2.32)

Theorem 2.3 [11]

Suppose $N \in C^p[a.b]$, *if* $N^{(m)}(s+u_0) = 0$ *for m=0, 1, 2, ..., p-1 and* $N^{(p)}(S+u_0) \neq 0$, *then the sequence* S_n *is of order p.*

Proof

By the hypothesis of theorem (2.3), from equation (2.32), we have:

$$S_{n+1}-S=\frac{1}{p!}N^{(p)}(S+u_0)(S_n-S)^p+\frac{1}{p+1!}N^{(p+1)}(S+u_0)(S_n-S)^{P+1}+\cdots \quad (2.33)$$

By dividing both sides of equation (2.33) by (S_n-S) we get:

$$\frac{S_{n+1}-S}{(S_n-S)^p}=\frac{1}{p!}N^{(p)}(S+u_0)+\frac{1}{p+1!}N^{(p+1)}(S+u_0)(S_n-S)+\cdots$$
(2.34)

Then we take the limit as $n\to\infty$ to both sides of equation (2.34):

$$\lim_{n\to\infty}\left|\frac{S_{n+1}-S}{(S_n-S)^p}\right|=\lim_{n\to\infty}\frac{1}{p!}N^{(p)}(S+u_0)+\lim_{n\to\infty}\frac{1}{p+1!}N^{(p+1)}(S+u_0)(S_n-S)+\cdots \quad (2.35)$$

Since $\lim_{n\to\infty}(S_n)=S$ then every terms that has (S_n-S) will be canceled so at the end we have:

$$\lim_{n\to\infty}\left|\frac{S_{n+1}-S}{(S_n-S)^p}\right|=\lim_{n\to\infty}\frac{1}{p!}N^{(p)}(S+u_0)=c \quad (2.36)$$

So by the previous definition the order of the sequence is p.

2.5 Formulation of Adomian polynomials for nonlinear cases

There are numerous cases of non-linear terms that the Adomian decomposition method may need to decompose and solve. The following cases show examples of non-linear terms decomposed by the Adomian decomposition method.

2.5.1 Polynomial non-linearities

The Adomian polynomials are easily given for the nonlinear term x_i^n in equation (2.9). As for other nonlinear terms such as $\sin x_3$, the first Adomian polynomials are formulated as:

$$A_0 = sinx_{30}$$

$$A_1 = x_{31}\cos(x_{30})$$

$$A_2 = \frac{1}{2!}x_{31}^2\sin(x_{30}) + x_{32}\cos(x_{30})$$

$$A_3 = \frac{1}{3!}x_{31}^3\,\mathrm{Cos}(x_{30}) + x_{32}x_{31}\sin(x_{30}) + x_{33}\cos(x_{30})$$

To further show this, another non-linear term is chosen which is $e^{-x_{10}x_{20}}$. The first Adomian polynomials for this term are formulated as follows:

$$A_0 = e^{-x_{10}x_{20}}$$

$$A_1 = -(x_{20}x_{11} + x_{21}x_{13})e^{-x_{10}x_{20}}$$

$$A_2 = \left(\frac{1}{2!}x_{11}^2x_{20}^2 + \frac{1}{2!}x_{21}^2x_{10}^2 - x_{11}x_{21} + x_{11}x_{21}x_{10}x_{20} - x_{21}x_{20} - x_{22}x_{10}\right)e^{-x_{10}x_{20}}$$

$$A_3 = \left(\frac{1}{3!}x_{11}^3x_{20}^3 - \frac{1}{3!}x_{21}^3x_{10}^3 + x_{11}^2x_{21}x_{20} - \frac{1}{2!}x_{11}^2x_{21}x_{10}x_{20}x_{20}^2 - x_{13}x_{20} - x_{23}x_{10} - x_{21}x_{12}x_{21}x_{12}x_{10}x_{20} - x_{11}x_{22} + x_{22}x_{10}^2x_{21} + x_{11}x_{22}x_{10}x_{20}x_{11}x_{12}x_{20}^2\right)e^{-x_{10}x_{20}}$$

2.5.2 Negative power non-linearities

When solving a negative power non-linear term i.e. a differential equation involving the term y^{-m}, where m is any positive integer. First five Adomian polynomials gained through the decomposition method are given as follows:

$$A_0 = y_0^{-m}$$

$$A_1 = -my_0^{-(m+1)}y_1$$

$$A_2 = \frac{1}{2!}m(m+1)y_0^{-(m+2)}y_1^2 - my_0^{-(m+1)}y_2$$

$$A_3 = -\frac{1}{3!}m(m+1)(m+2)y_0^{-(m+3)}y_1^3 + m(m+1)y_0^{-(m+2)}y_1y_2 - my_0^{-(m+1)}y_3$$

$$\begin{aligned} A_4 = -\frac{1}{4!}&(-3-m)(-2-m)(-1-m)my_0^{-4-m}y_1^4 \\ &-\frac{1}{2!}(-2-m)(-1-m)my_0^{-3-m}y_1^2y_2 - \frac{1}{2!}(-1-m)my_0^{-2-m}y_2^2 \\ &- 24(-1-m)my_0^{-2-m}y_1y_3 - my_0^{-1-m}y_4 \end{aligned}$$

$$\begin{aligned} A_5 = -\frac{1}{5!}&(-4-m)(-3-m)(-2-m)(-1-m)my_0^{-5-m}y_1^5 \\ &-\frac{1}{3!}(-3-m)(-2-m)(-1-m)my_0^{-4-m}y_1^3y_2 \\ &-\frac{1}{2!}(-2-m)(-1-m)my_0^{-3-m}y_2y_2^2 \\ &-\frac{1}{2!}(-2-m)(-1-m)my_0^{-3-m}y_1^2y_3 - (-1-m)my_0^{-2-m}y_2y_3 \\ &- (1-m)my_0^{-2-m}y_1y_4 - my_o^{-1-m}y_5 \end{aligned}$$

2.5.3 Decimal power non-linearities

Another form on non-linear terms are decimal power terms i.e. terms with $x^{\frac{1}{m}}$, where *m* is any integer. The first few Adomian polynomials for a decimal power non-linear term $y^{\frac{1}{3}}$ are formulated as follows:

$$A_0 = y_0^{1/3}$$

$$A_1 = \frac{y_1}{3} y_0^{2/3}$$

$$A_2 = \frac{1}{2!}\left(-\frac{2}{9}\frac{y_1^2}{y_0^{5/3}} + \frac{y_2}{y_0^{3/2}}\right)$$

$$A_3 = \frac{1}{3!}\left(\frac{10}{27}\frac{y_1^3}{y_0^{8/3}} - \frac{4}{3}\frac{y_1 y_2}{y_0^{5/3}} + 2\frac{y_3}{y_0^{2/3}}\right)$$

$$A_4 = \frac{1}{4!}\left(\frac{80}{81}\frac{y_1^4}{y_0^{11/3}} - \frac{40}{9}\frac{y_1^2 y_2}{y_0^{8/3}} - \frac{8}{3}\frac{y_2^2}{y_0^{2/3}} - \frac{16}{3}\frac{y_1 y_2}{y_0^{5/3}} + 8\frac{y_4}{y_0^{2/3}}\right)$$

$$A_5 = \frac{1}{5!}\left(\frac{880}{243}\frac{y_1^5}{y_0^{14/3}} - \frac{1600}{81}\frac{y_1^3 y_2}{y_0^{11/3}} - \frac{200}{9}\frac{y_1 y_2^2}{y_0^{8/3}} + \frac{200}{9}\frac{y_1^2 y_3}{y_0^{8/3}} - \frac{80}{3}\frac{y_2 y_3}{y_0^{5/3}}\right.$$
$$\left. - \frac{80}{3}\frac{y_1 y_4}{y_0^{5/3}} + 40\frac{y_5}{y_0^{2/3}}\right)$$

2.6 Boundary conditions

For boundary value problem, the integration constants require evaluation for the Adomian decomposition method. For the Adomian decomposition method, there are three approaches for implementing the boundary conditions:

1. The zeroth order intake all boundary conditions, while other orders have homogeneous boundary conditions.
2. Obtain the solution to the desired order and then evaluate the constants.
3. Evaluate the constants at every order.

While the first approach is the easiest to implement, it is generally not recommended due to the nature of the decomposition. Since the method chooses to attach boundary conditions to the base solution, this method will behave as a regular perturbation thus hindering its ability of capture the essential features of the problem without a small parameter. As for the second approach, while it seems to be a valid alternative, it only works only for linear operators. For non-linear operators, the constants would need to be determined by solving a polynomial of order *n*. To do this, a numerical technique that outweighs the benefits of an analytical model is needed. Finally, the only option left is the option that demands the evaluation of the integration constants at every order of the solution. The boundary conditions implemented by this approach are implemented with a relatively easy framework, however, it consists of several evaluations at each order.

2.7 Applications of the (ADM) method

There are a wide range of equations that can be solved by Adomian decomposition method, this includes algebraic equations, ordinary and partial differential equations, integral equations, and integral differential equations. As for its applications, the (ADM) method has received extensive applications in multiple fields and their disciplines, e.g. physics, engineering, chemistry, chaos theory, heat and mass transfer, etc…. In the field of fluid mechanics, there are several problems that utilize the (ADM) method. Examples of these studies include Bulut et al.'s study [13] that used the

(ADM) method to solve the governing Navier-Stokes equations of “a steady flow problem of a viscous incompressible fluid through an orifice” analytically. Results of the study show that (ADM) method results are reliable and practical method and require less computational work when compared to results of a numerical solution. Another study conducted by Momani and Odibat also applied the (ADM) method to “a time-fractional Navier-Stokes equation” for “unsteady flow of a viscous fluid in a tube” [21]. The study concluded that since the time-fractional Navier-Stokes equations are non-linear, a known general method to solve these equations doesn’t exist and an exact solution can only be obtained for a very limited number of cases. The study also shows that the (ADM) method allows for the construction of an analytic solution in the form of a series through a reliable technique that requires less work than the traditional methods used. The study finally concluded that the solution depended continuously on the time fractional derivative. A more recent study conducted by Wang used the (ADM) method to the classical Blasius equation [23]. Using this method, Wang was able to easily provide an analytical solution to this classical problem. However, the value of the parameter $y''(0)$ was impossible to determine with this solution. To overcome this, the problem was transformed into a singular nonlinear boundary value problem and the (ADM) method was applied to it. By using this method the parameter $y''(0)$ was obtained easily as well as a 5-term approximate solution comparable to the numerical solution. This study provided further proof of the (ADM) method ability to provide reliable solutions. As a finale example to the application of (ADM) method, a study by Al-Hayani and Casu´s will be used. In their study, the (ADM) method was applied to first order initial value problems with Heaviside functions and other discontinuities [9]. The analysis worked well with the (ADM) method and multiple findings were reached. This includes that the size of the jump had virtually no effect on the method convergence and that some cases needed more digits included to avoid any unstable oscillation. At the end, Al-Hayani and Casu´s both concluded that the error could be reduced by modifying the (ADM) method slightly to include the term associated with the inverse operator applied to the source function in the first Adomian polynomial instead of the initial term in the series solution.

2.8 Advantages and Disadvantages of the (ADM) method

The reason for the multiple application of (ADM) method can be credited to its many advantages. The (ADM) method's main advantage would be its requirement of less computational work than traditional methods, as shown by multiple studies [18] [23] [24]. The method's ability to solve nonlinear problems without linearization is another advantage that elevates the (ADM) method in the eyes of many researchers. Wang mentioned in his study mentioned earlier that the (ADM) method can handle nonlinearities which are "quite general" and generates solutions that are more realistic than solutions achieved via model simplification that are required by other techniques. A study by Wazwaz states that "The main advantage of the method is that it can be applied directly for all types of differential and integral equations, linear or nonlinear, homogeneous or inhomogeneous, with constant coefficients or with variable coefficients [24]. Another important advantage is that the method is capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution." A study by Jiao et al. clearly states that the "ADM is quantitative rather than qualitative, analytic, requiring neither linearization nor perturbation, and continuous with no resort to discretization and consequent computer-intensive calculations"

[20]. Another advantage would be the method's ability to develop a reliable analytical solution. Jiao et al also states some of the disadvantages of the ADM method by stating: "although the series can be rapidly convergent in a very small region, it has very slow convergence rate in the wider region and the truncated series solution is an inaccurate solution in that region, which will greatly restrict the application area of the method.". This means that the (ADM) method must be truncated for practical application since it gives a series solution and that the rate and region of convergence are a possible disadvantage of the method. However, their claim requires further investigation before it can be fully accepted by the science community in large.

Chapter Three

Adomian decomposition of a system of nonlinear equations

Adomian decomposition of a system of nonlinear equations

In this chapter, the Adomian decomposition of a system of nonlinear equations is presented along with the solution method of these systems. This chapter also presents the mathematical analysis of this method and accompanying theorem and proof. Numerical examples showing the solution of systems of nonlinear equations will also be presented.

3.1 Decomposition method

The first attempt to solve nonlinear equations using the (ADM) method was by K. Abboi and Y. Cherrault in 1994 [1]. In their study they applied the ADM method to solve the equation $f(x) = 0$, where $f(x)$ is a nonlinear function. This method was also used by Babolian et al to solve a system of linear equations as well as an equivalent of this method using the classical iterative method Jacobi [11]. This method can also be extended to solve a system of nonlinear equations. To show this, consider the following nonlinear system of equations:

$$\begin{gathered} f_1(x_1, x_2, \dots, x_n) = 0 \\ f_2(x_1, x_2, \dots, x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \dots, x_n) = 0 \end{gathered} \tag{3.1}$$

where each f_i function maps a vector $X = (x_1, x_2, \dots, x_n)^t$ of the n-dimensional space $\mathbb{R}^n$ into the real numbers $\mathbb{R}$. It is assumed that the previous system admits a unique solution. Now, consider i^{th} equation of this system:

$$f_i(x_1, x_2, \dots, x_n) = 0 \tag{3.2}$$

Without losing generality, x_i can be obtained from equation (3.2) in canonical form as follows:

$$x_i = c_i + g_i(x_1, x_2, \dots, x_n) \tag{3.3}$$

where $g_i(x_1, x_2, \dots, x_n)$ is a nonlinear function and c_i is a constant. In order to apply the ADM method, let

$$x_i = \sum_{m=0}^{\infty} x_{im} \tag{3.4}$$

and

$$g_i(x_1, x_2, \dots, x_n) = \sum_{m=0}^{\infty} A_{im} \tag{3.5}$$

where A_{im}'s are the Adomian polynomials depending on $x_{10}, \dots, x_{1m}, \dots, x_{n0}, \dots, x_{nm}$ [2]. By substituting equations (3.4) and (3.5) in equation (3.3), it becomes:

$$\sum_{m=0}^{\infty} x_{im} = c_i + \sum_{m=0}^{\infty} A_{im} \tag{3.6}$$

From the equation(3.6), it can be defined that:

$$\begin{aligned} x_{i0} &= c_i \\ x_{i,m+1} &= A_{im}, \quad i = 1, \dots, n, m = 0, 1, 2, \dots \end{aligned} \tag{3.7}$$

The next step is, to approximate x_i by

$$\varphi_{ik} = \sum_{m=0}^{k-1} x_{im} \tag{3.8}$$

where

$$\lim_{k \to \infty} \varphi_{ik} = x_i \tag{3.9}$$

To determine the Adomian polynomials, the following formula is used [2]:

$$A_{im}(x_{10},\dots,x_{1m},\dots,x_{n0},\dots,x_{nm}) = \frac{1}{m!}\left[\frac{d^m}{d\lambda^m} g_i(x_1,\dots,x_n)\right]_{\lambda=0} \quad (3.10)$$

Using the following equation:

$$x_{i\lambda} = \sum_{j=0}^{m} \lambda^j x_{ij}, \qquad i = 1,\dots,n \qquad (3.11)$$

The following results can be derived:

$$A_{i0}(x_{10},x_{20},\dots,x_{n0}) = g_i(x_{10},x_{20},\dots,x_{n0})$$

$$A_{im}(x_{10},\dots,x_{1m},x_{20},\dots,x_{2m},\dots,x_{n0},\dots,x_{nm}) =$$
$$\sum\nolimits_{\Omega}\left(\frac{x_{11}^{k_{11}}}{k_{11}!}\cdots\frac{x_{m1}^{k_{m1}}}{k_{m1}!}\right)\left(\frac{x_{12}^{k_{12}}}{k_{12}!}\cdots\frac{x_{m2}^{k_{m2}}}{k_{m2}!}\right)\cdots\left(\frac{x_{1n}^{k_{1n}}}{k_{1n}!}\cdots\frac{x_{mn}^{k_{mn}}}{k_{mn}!}\right)\times$$
$$\frac{\partial^{\Omega_1+\Omega_2+\cdots\Omega_n}}{\partial x_1^{\Omega_1}\partial x_2^{\Omega_2}\dots x_n^{\Omega_n}} g_i(x_{10},x_{20},\dots,x_{n0}), \quad m \neq 0 \qquad (3.12)$$

where Ω represents:

$$(k_{11}+2k_{21}+\cdots+mk_{m1})+\cdots+(k_{1m}+2k_{2m}+\cdots+mk_{mn}) = m \quad (3.13)$$

and

$$\Omega_i = k_{1i}+k_{2i}+\cdots+k_{mi}, \qquad i = 1,\dots,n \qquad (3.14)$$

3.2 Convergence of the method

Take into consideration the system of equations (3.1), the solution that is required is in the family:

$$x_{i\lambda} = \sum_{j=0}^{\infty} x_{ij}\lambda^j, \qquad i = 1,2,\dots,n. \qquad (3.15)$$

Let $\rho = \min\{\rho_1, \rho_2, \dots, \rho_n\}$, where ρ_i is the convergence radius of the series (3.15), assuming that ρ>1. By following Y. Cherrault and Y. Saccomandi's study [14], and extending to n-dimensional space, equation (3.15) converges for $|\lambda| \leq \rho$, with $\rho > 1$.

By supposing that $g_{i\lambda}(x_1, x_2, \dots, x_n)$ can be expanded in an entire series, we obtain the following equation:

$$g_{i\lambda}(x_1, x_2, \dots, x_n) = \sum_{m=0}^{\infty} \sum_{\substack{k_1+k_2+\cdots k_n=m \\ k_1,\dots,k_n \in W=\{0,1,\dots\}}} a_{k_1 k_{2,\dots,}k_n} x_1^{k_1} x_1^{k_2}, \dots, x_n^{k_n} \quad (3.17)$$

with convergence radius $\rho^* > 1$. The previous equation implies that the series in equation (3.15) converges for $||x|| < \rho^*$, With ρ^*>1. By using an extension of a classical results that are given by L. Gabet [17] and substituting equation (3.15) into (3.17), the following series is obtained:

$$\sum_{m=0}^{\infty} C_m \lambda^m \quad (3.18)$$

which has a convergence radius that is strictly greater than 1:

$$g_{i\lambda}(x_1, x_2, \dots, x_n) == \sum_{m=0}^{\infty} \sum_{\substack{k_1+k_2+\cdots k_n=m \\ k_1,\dots,k_n \in W=\{0,1,\dots\}}} a_{k_1 k_{2,\dots,}k_n} \prod_{i=1}^{n} \left(\sum_{j=0}^{\infty} x_{ij} \lambda^j \right)^{k_i} \quad (3.19)$$

The m-row of this array converges to A_{im} defined in equation (3.10) by setting $\lambda = 1$, because $g_{i\lambda}(x_1, x_2, \dots, x_n)$ can be developed in a Taylor series. The next problem is to prove the convergence of the double series in equation (3.19) for $\lambda = 1$, which is given in the following theorem.

Theorem 3.1

If $g_{i\lambda}(x_1, x_2, \dots, x_n)$ is an analytic function of n variables $x_1, x_2, \dots, x_n$ in $||x|| < R$ and x_i , i=1, 2, …, n, can be decomposed as an infinite series $x_i = \sum x_{im}$, the parameterization $x_{i\lambda} = \sum x_{im}\lambda^m$ is absolutely convergent for $\lambda \in [-1, 1]$ and the series x_i can be majored by:

$$\frac{m'}{n(1+\varepsilon)}\left(1 + \frac{1}{1+\varepsilon}\left(\frac{\lambda}{\rho}\right) + \cdots + \frac{1}{(1+\varepsilon)^n}\left(\frac{\lambda}{\rho}\right)^n + \cdots\right) \tag{3.20}$$

where $m' \geq M$, (M is the upper limit for the x_i), $(M/R) < \varepsilon$ and $\rho \geq 1$, then the double series converges for $\lambda=1$.

Proof

$g_{i\lambda}(x_1, \dots, x_n)$ is analytic in $||x|| < R$, then it can be written as:

$$g_{i\lambda}(x_1, \dots, x_n) \leq L\left(1 + n\left(\frac{x}{R'}\right) + \cdots + n^n\left(\frac{x}{R'}\right)^n + \cdots\right), \tag{3.21}$$

where $||g_{i\lambda}(x_1, \dots, x_n)||^* \leq L$ ($||\ \ ||^*$ is the dual norm), and $x = \max\{x_1, \dots, x_n\}$ and $R' \in [M, R]$. By employing the hypothesis in equation (3.20), we obtain:

$$x \leq \frac{m'}{n(1+\varepsilon)}\left(1 + \frac{1}{1+\varepsilon}\left(\frac{\lambda}{\rho}\right) + \cdots + \frac{1}{(1+\varepsilon)^n}\left(\frac{\lambda}{\rho}\right)^n + \cdots\right) =$$

$$\frac{m'}{n(1+\varepsilon)}\left(\frac{1}{1-\frac{1}{1+\varepsilon}\left(\frac{\lambda}{\rho}\right)}\right) = \frac{m'}{n\left(1+\varepsilon-\frac{\lambda}{\rho}\right)} \tag{3.22}$$

Substituting (3.22) into (3.21), the following equation is obtained:

$$g_{i\lambda}(x_i, \dots, x_n) \leq L\left[1 + \frac{m'}{R'\left[1+\varepsilon-\left(\frac{\lambda}{\rho}\right)\right]} + \left(\frac{m'}{R'\left[1+\varepsilon-\left(\frac{\lambda}{\rho}\right)\right]}\right)^2 + \cdots + \left(\frac{m'}{R'\left[1+\varepsilon-\left(\frac{\lambda}{\rho}\right)\right]}\right)^n + \cdots\right] \tag{3.23}$$

To reach convergence for equation (3.21), the following condition must be achieved:

$$\frac{m'}{R'\left[1+\varepsilon-\left(\frac{\lambda}{\rho}\right)\right]} < 1 \tag{3.24}$$

i.e.

$$\lambda < \rho\left(1 + \varepsilon - \frac{M}{R'}\right) \tag{3.25}$$

By choosing $\rho = 1$, then equation (3.21) converges for $\lambda = 1$, if $\varepsilon > (M/R')$.

3.3 Solving system of non-linear equations

To show the effectiveness of using the Adomian decomposition method in solving systems of non-linear equations, the two following non-linear systems will be solved. The first system is as follows:

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0$$
$$x_1x_2^2 + x_1 - 10x_2 + 8 = 0$$

Using equation (3.6) involves the following:

$$x_1 = \sum_{m=0}^{\infty} x_{1m} = \frac{8}{10} + \frac{1}{10}x_1^2 + \frac{1}{10}x_2^2 x_1$$

$$= \frac{8}{10} + \frac{1}{10}\sum_{m=0}^{\infty} A_{1m}(x_1^2) + \frac{1}{10}\sum_{m=0}^{\infty} A_{1m}(x_2^2)$$

$$x_2 = \sum_{m=0}^{\infty} x_{2m} = \frac{8}{10} + \frac{1}{10}x_1x_2^2 + \frac{1}{10}x_1$$

$$x_2 = \frac{8}{10} + \frac{1}{10}\sum_{m=0}^{\infty} A_{2m}(x_1x_2^2) + \frac{1}{10}\sum_{m=0}^{\infty} A_{2m}(x_1)$$

The $A_{im}(x^n)$ are given by:

$$A_{i0}(x^n) = x_{i0}^n$$

$$A_{i1}(x^n) = nx_{i0}^{n-1}x_{i1}$$

$$A_{i2}(x^n) = \frac{1}{2}n(n-1)x_{i0}^{n-2}x_{i1}^2 + nx_{i0}^{n-1}x_{i2}$$

$$A_{i3}(x^n) = \frac{1}{6}n(n-1)(n-2)x_{i0}^{n-3}x_{i1}^3 + n(n-1)x_{i0}^{n-2}x_{i1}x_{i2} + nx_{i0}^{n-1}x_{i3}$$

$$A_{i4}(x^n) = \frac{1}{24}n(n-1)(n-2)(n-3)x_{i0}^{n-4}x_{i1}^4 + \frac{1}{2}n(n-1)(n-2)x_{i0}^{n-3}x_{i1}^2x_{i2} + n(n-1)x_{i0}^{n-2} \times \left(\frac{1}{2}x_{i2}^2 + x_{i1}x_{i3}\right) + nx_{i0}^{n-1}x_{i4}$$

and the first few Adomian polynomials for the nonlinear term $x_1x_2^2$ are formulated as:

$$A_0 = x_{10}x_{20}^2$$

$$A_1 = 2x_{10}x_{20}x_{21} + x_{11}x_{20}^2$$

$$A_2 = x_{10}x_{21}^2 + 2x_{10}x_{20}x_{22} + 2x_{11}x_{20}x_{21} + x_{12}x_{20}^2$$

$$A_3 = 2x_{10}x_{20}x_{23} + 2x_{10}x_{21}x_{22} + x_{11}x_{21}^2 + 2x_{11}x_{20}x_{22} + 2x_{12}x_{20}x_{21} + x_{13}x_{20}^2$$

$$A_4 = (2x_{20}x_{24} + 2x_{21}x_{23} + x_{22}^2)x_{10} + (2x_{20}x_{23} + 2x_{21}x_{22})x_{11} + (x_{21}^2 + 2x_{20}x_{22})x_{12} + 2x_{20}x_{21}x_{13} + x_{20}^2x_{14}$$

From equation (3.7), we obtain:

$x_{10} = 0.80000001$

$x_{11} = 0.1414400$

$x_{12} = 0.0360233$

$x_{13} = 0.0127908$

$x_{14} = 0.00520737$

$x_{15} = 0.00231817$

$x_{20} = 0.88000001$

$x_{21} = 0.076096$

$x_{22} = 0.0252698$ $x_{24} = 0.00441812$

$x_{23} = 0.0998425$ $x_{25} = 0.00208511$

Sum of the first five terms gives:

$$\varphi_{15} = x_{10} + \cdots + x_{14} = 0.99778$$

$$\varphi_{25} = x_{20} + \cdots + x_{24} = 0.997853$$

which is a good approximation of the exact solution $x = (1,1)^t$. To further show the capabilities of the Adomian decomposition method take the following non-linear system of equations:

$$\begin{aligned} 15x_1 + x_2^2 - 4x_3 &= 13 \\ x_1^2 + 10x_2 - e_3^{-x} &= 11 \\ x_2^3 - 25x_3 &= -22 \end{aligned}$$

Using equation (3.6) involves the following:

$$x_1 = \sum_{m=0}^{\infty} x_{1m} = \frac{13}{15} - \frac{1}{15}x_2^2 + \frac{4}{15}x_3$$

$$x_1 = \frac{13}{15} - \frac{1}{15}\sum_{m=0}^{\infty} A_{1m}(x_2^2) + \frac{4}{15}\sum_{m=0}^{\infty} A_{1m}(x_3)$$

$$x_2 = \sum_{m=0}^{\infty} x_{2m} = \frac{11}{10} + \frac{1}{10}x_1^2 + \frac{1}{10}e^{-x_3}$$

$$x_2 = \frac{11}{10} - \frac{1}{10}\sum_{m=0}^{\infty} A_{2m}(x_1^2) + \frac{1}{10}\sum_{m=0}^{\infty} A_{2m}(e^{-x_3})$$

$$x_3 = \sum_{m=0}^{\infty} x_{3m} = \frac{22}{25} + \frac{1}{25}x_2^3$$

$$x_3 = \frac{22}{25} - \frac{1}{25}\sum_{m=0}^{\infty} A_{3m}(x_2^3)$$

The polynomials $A_{im}(x^n)$ are obtained the same way as the previous example and for the non-linear term e^{-x_i} , we have:

$$A_{im}(e^{-x_i}) = (-1)^{m+1}\frac{m^{m-1}}{m!}e^{-mx_{i0}}$$

From equation (3.7), we obtain:

$$x_{10} = 0.86666667$$

$$x_{11} = 0.15400000$$

$$x_{12} = 0.01913015$$

$$x_{13} = 0.00506067$$

$$x_{14} = -0.00262029$$

$$x_{20} = 1.10000002$$

$$x_{21} = -0.3363282$$

$$x_{22} = -0.04389782$$

$$x_{23} = 0.00501670$$

$$x_{24} = -0.01179828$$

$$x_{30} = 0.88000000$$

$$x_{31} = 0.05324000$$

$$x_{32} = -0.0488349$$

$$x_{33} = 0.00632872$$

$$x_{34} = -0.00111667$$

Sum of the first six terms gives:

$$\varphi_{15} = x_{10} + \cdots + x_{15} = 1.04215$$

$$\varphi_{25} = x_{20} + \cdots + x_{25} = 1.03109$$

$$\varphi_{35} = x_{30} + \cdots + x_{35} = 0.923848$$

Which is an approximation of the exact solution $x = (1.04214966, 1.03109169, 0.92384809)^t$.

Conclusion

In this study, the use of the Adomain decomposition method to solve systems of non-linear equation was studied in details and evaluated. Based on the results of this study, it was determined that the Adomain decomposition method is valid choice in solving a system of non-linear equations. During numerical testing, the Adomain Decomposition method provided accurate results with good approximation.

الملخص: -

تناقش هذه الدراسة احدى الطرق الموحدة المستخدمة في حل المعادلات الخطية وغير الخطية وكذلك المعادلات التفاضلية العادية والجزئية وتعرف هذه الطريقة بطريقة أدومين التحليلية. وفي هذه الدراسة سيتم مناقشة طريقة أدومين بالتفصيل ليتضمن المعلومات الأساسية وكثيرات الحدود والتقارب والشروط الحدية والاستخدامات والمميزات. وعندما يتم مناقشة طريقة أدومين بشكل مفصل سيتم مناقشة تطبيقاتها على منظومة المعادلات غير الخطية وسيتم إعطاء أمثلة عددية لتوضيح مدى فاعلية هذه الطريقة. النتائج من خلال الأمثلة العددية توضح مدى فاعلية طريقة أدومين التحليلية في حل منظومة المعادلات غير الخطية بسهولة وبدقة مرغوبة.

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وزارة التعليم العالي والبحث العلمي

جامعة الزاوية

إدارة الدراسات العليا والتدريب

كلية العلوم

قسم الرياضيات

طريقة تحليل أدومين لحل منظومة واسعة من المعادلات غير الخطية

رسالة مقدمة كجزء من متطلبات نيل الإجازة العالية "الماجستير" في علم الرياضيات

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