Numerical Study of Electromagnetic Waves by using Finite Difference Time Domain (FDTD) Technique in Two and Three Dimensions

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Abstract

In this paper, the finite difference time domain (FDTD) method applied for solving the time dependent Maxwell's curl equations in two dimensions (2D) and three dimensions (3D) systems. In the FDTD, the computational domain is terminated with the absorbing boundary conditions (ABCs) that applied in two dimensional (2D-FDTD) and three dimensional (3D-FDTD). Two dimensions system is used as the transverse magnetic (TM) wave propagates in free space. We are

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constructed the obstacles in the centre of a domain for examples the cube, cylinder shapes and wire with the purpose of acting as the perfect electric conductors (PECs). The electric and magnetic fields computed in space at every time step and the electromagnetic wave interacted with PECs. The results of simulations demonstrated that there are no signals appeared in the regions of the obstacles. Because the electric fields components are equal to zeros while outside the obstacles the signals appeared and updated at each cell in the domain. This can be obtained in the simulations when performed in 2D and 3D. It can be noted that the similar results can be clearly observed. In order to make a comparison between simulations, the distributions of electromagnetic waves affected when added for example wire compare to the simulation without wire in the domain. Moreover, we obtained very good simulations results by using the absorbing boundary conditions. Because the ABCs absorbed the electromagnetic waves and minimized the reflections that come from the boundaries.

Key words: *Maxwell's equations, Finite difference time domain (FDTD) method, transverse magnetic wave (TM), first and second orders Mur's absorbing boundary conditions (ABCs), perfect electric conductor (PEC).*

1. Introduction

There are many problems in electromagnetic field which is sometimes difficult to solve and find an exact solution analytically. To overcome this difficulty, many numerical methods have been developed to simulate the electromagnetic such as the method of moment (MoM), the finite element (FEM) and the finite difference time domain (FDTD). The FDTD solves Maxwell's equations directly. The FEM solves the vector wave equation derived from Maxwell's equations while the MoM solves an integral equation [1]. The FDTD is one of the most popular method and widely used for studying the propagation of electromagnetic waves. The FDTD is firstly introduced by Kane. S. Yee in a paper published in 1966 [2]. Nowadays, this technique is applied in many applications to solve Maxwell's equations numerical such as computing the specific absorption rate distribution [3]. In FDTD simulation, there is a need of absorbing boundary conditions (ABCs) and becomes necessary to terminate a domain. Because the ABCs should be utilized to obtain good numerical results by simulating the infinite space that surrounds the finite computational domain. This will lead to reduce as much as possible the reflections come from the edges then achieving an acceptable approximation of the solution. The ABCs based on the Mur's approach which is used as the first and second orders that applied in three dimensional (3D-FDTD) and two dimensional (2D-FDTD), respectively [4]. The aim of this research is to simulate the propagation of electromagnetic waves in two and three dimensions when inserting the same obstacles in the space in order to make a comparison between the distributions of electromagnetic waves and also determine the type of the shape as constructed in different shapes in a domain as well as study the propagation of sinusoidal waves when striking the perfect electric conductor (PEC) and observe the behaviour of the electromagnetic waves. We expect that the signals will not penetrate inside the obstacles and the calculation of the distributions will explain that the signals will reflect and diffract by the obstacles in the space.

2. Method

The FDTD method is utilized to solve Maxwell's equations. The equations can be written in a general form as the following [2]:

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_o} \nabla \times \mathbf{H}$$
(1.a)

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_o} \nabla \times \mathbf{E} \qquad (1.b)$$

Where the **E** is electric field, **H** is magnetic field, ε_o is permittivity and μ_o is the permeability of free space.

It can be written Maxwell's equations in three-dimensional system as the electric and magnetic field components as the following [5]:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_o} (\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}) \qquad (2.a)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_o} (\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}) \qquad (2.b)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_o} (\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \qquad (2.c)$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu_o} (\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}) \qquad (2.d)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_o} (\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) \qquad (2.e)$$

$$\frac{\partial H_z}{\partial t} = -\frac{1}{\mu_o} (\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}) \qquad (2.f)$$

Applying the central finite difference approximation method for the space and time derivatives into the equation (2.a-2.f) by using these equations:

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+\frac{1}{2}}(i,j,k) - F^{n-\frac{1}{2}}(i,j,k)}{\Delta t} \qquad (3.a)$$

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n(i+\frac{1}{2},j,k) - F^n(i-\frac{1}{2},j,k)}{\delta x} \qquad (3.b)$$

Therefore, six discretization updating equations in three-dimensional can be generated which will be used in the simulations [5]: firstly, the electric fields components can be written as:

$$E_x|_{i+\frac{1}{2},j,k}^{n+1} = E_x|_{i+1/2\,,j,k}^{n} + \frac{1}{\varepsilon_o}.\left(\frac{\Delta t}{\delta}\left(H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}\right) - \frac{\Delta t}{\delta}\left(H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}\right)\right) \quad (4.a)$$

$$E_y|_{i,j+\frac{1}{2},k}^{n+1} = E_y|_{i\,,j+1/2,k}^{n} + \frac{1}{\varepsilon_o}.\left(\frac{\Delta t}{\delta}\left(H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j+\frac{1}{2},k-\frac{1}{2}}^{n+\frac{1}{2}}\right) - \frac{\Delta t}{\delta}\left(H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i-\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}}\right)\right) \quad (4.b)$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n+1} = E_z|_{i\,,j,k+1/2}^{n} + \frac{1}{\varepsilon_o}.\left(\frac{\Delta t}{\delta}\left(H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}\right) - \frac{\Delta t}{\delta}\left(H_x|_{i\,,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}}\right)\right) \quad (4.c)$$

And the magnetic fields components are:

$$H_x|_{i\,,j+1/2,k+1/2}^{n+1/2} = H_x|_{i\,,j+1/2,k+1/2}^{n-1/2} + \frac{1}{\mu_o}.\left(\frac{\Delta t}{\delta}\left(E_z|_{i\,,j,k+1/2}^{n} - E_z|_{i\,,j+1,k+1/2}^{n}\right) - \frac{\Delta t}{\delta}\left(E_y|_{i\,,j+1/2,k}^{n} - E_y|_{i\,,j+1/2,k+1}^{n}\right)\right) \quad (4.d)$$

$$H_y|_{i+1/2\,,j,k+1/2}^{n+1/2} = H_y|_{i+1/2\,,j,k+1/2}^{n-1/2} + \frac{1}{\mu_o}.\left(\frac{\Delta t}{\delta}\left(E_z|_{i+1\,,j,k+1/2}^{n} - E_z|_{i\,,j,k+1/2}^{n}\right) - \frac{\Delta t}{\delta}\left(E_x|_{i+\frac{1}{2},j,k+1}^{n} - E_x|_{i+1/2\,,j,k}^{n}\right)\right) \quad (4.e)$$

$$H_z|_{i+1/2\,,j+1/2,k}^{n+1/2} = H_z|_{i+1/2\,,j+1/2,k}^{n-1/2} + \frac{1}{\mu_o}.\left(\frac{\Delta t}{\delta}\left(E_x|_{i+1/2,j+1,k}^{n} - E_x|_{i+1/2,j,k}^{n}\right) - \frac{\Delta t}{\delta}\left(E_y|_{i+1,j+1/2,k}^{n} - E_y|_{i\,,j+1/2,k}^{n}\right)\right) \quad (4.f)$$

Moreover, we can consider Maxwell's equations in two dimensions and the equations into two-dimensional system can be reduced by assuming that $\frac{\partial}{\partial z} = 0$, as a result, the 2D-FDTD equations can be obtained and classified into two groups [2]:

Transverse Electric wave (TE):

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_o}\frac{\partial H_z}{\partial y} \quad (5.a)$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon_o}\frac{\partial H_z}{\partial x} \quad (5.b)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu_o}(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}) \quad (5.c)$$

Transverse Magnetic wave ™:

$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon_o}(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) \quad (6.a)$$

$$\frac{\partial H_x}{\partial \mathrm{t}} = -\frac{1}{\mu_o}\frac{\partial \mathrm{E_z}}{\partial \mathrm{y}} \quad (6.b)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_o}\frac{\partial E_z}{\partial x} \quad (6.c)$$

The two modes of electromagnetic waves are characterized in two-dimensional as the first is called transverse electric wave (TE) or TE to *z* polarization and second is called transverse magnetic wave (TM) or TM to *z* polarization. When applying the central finite difference approximation method into equation (6), we will obtain 2D-FDTD updating equations for the TM wave in two dimensions:

$$H_x^{n+\frac{1}{2}}(i,j+\tfrac{1}{2}) = H_x^{n-\frac{1}{2}}(i,j+\tfrac{1}{2}) - \frac{\delta t}{\mu_0\,\delta}(E_z^n(i,j+1) - E_z^n(i,j)) \quad (7.a)$$

$$H_y^{n+\frac{1}{2}}\left(i+\tfrac{1}{2},j\right) = H_y^{n-\frac{1}{2}}\left(i+\tfrac{1}{2},j\right) + \frac{\delta t}{\mu_0\,\delta}(E_z^n(i+1,j) - E_z^n(i,j) \quad (7.b)$$

$$E_z^{n+1}(i,j) = E_z^n(i,j) + \frac{\delta t}{\varepsilon_{o\,\delta}}\left(H_y^{n+\frac{1}{2}}\left(i+\tfrac{1}{2},j\right) - H_y^{n+\frac{1}{2}}\left(i-\tfrac{1}{2},j\right)\right) - \frac{\delta t}{\varepsilon_{o\,\delta}}\left(H_x^{n+\frac{1}{2}}(i,j+\tfrac{1}{2}) - H_x^{n+\frac{1}{2}}(i,j-\tfrac{1}{2})\right) \quad (7.c)$$

Furthermore, we should use a good and the efficient boundary to absorb electromagnetic wave at the walls. The boundary should be implemented in the FDTD programs to absorb electromagnetic waves at the end of a computational domain. There is an absorbing boundary condition which is called the Mur's absorbing boundary condition (ABC), the boundary conditions published in a paper by G. Mur [4]. Nowadays, many researchers are still using this ABC to minimize any reflections when the signals reach at the edges. Therefore, implementations of first order Mur's absorbing boundary condition in 3D-FDTD system, at the boundary x=0 is:

$$E_z^{n+1}\left(0,j,k+\frac{1}{2}\right) = E_z^{n}\left(1,j,k+\frac{1}{2}\right) + \left(\frac{c\delta t-\delta}{c\delta t+\delta}\right)\left(E_z^{n+1}\left(1,j,k+\frac{1}{2}\right) - E_z^{n}\left(0,j,k+\frac{1}{2}\right)\right) \quad (8)$$

In three-dimensional FDTD system should be added an absorbing boundary condition at the walls. Therefore, the discretization equations similar to equation (8) will be implemented in the program to minimize reflection in order to limit the area of computation. Moreover, we implemented the Mur's second order absorbing radiation boundary condition for two dimensions system and a square grid used as $\Delta x = \Delta y = \delta$, the Mur ABC can be written as the example at the boundary x=0 for E_z discretized:

$$E_z^{n+1}(0,j) = -E_z^{n-1}(1,j) + \frac{(c\,\delta t-\delta)}{(c\,\delta t+\delta)}\left(E_z^{n+1}(1,j) + E_z^{n-1}(0,j)\right) + \frac{2\delta}{c\,\delta t+\delta}\left(E_z^{n}(0,j) + E_z^{n}(1,j)\right) + \frac{(c\,\delta t)^2}{2\delta(c\,\delta t+\delta)}\left(E_z^{n}(0,j+1) + E_z^{n}(0,j-1) - 2E_z^{n}(0,j) + E_z^{n}(1,j+1) + E_z^{n}(1,j-1) - 2E_z^{n}(1,j)\right) \quad (9)$$

The above equation is the FDTD update Mur's second order boundary condition (ABCs). There are equations similar to equation (9) that will be applied to all walls in the program to avoid the reflections. This boundary can be used as absorbing boundary conditions (ABCs) to make the computational domain as an open as possible to reduce any reflections back, the ABCs will simulate the infinite space by absorbing the EM wave at the boundaries. This will be explained in the simulations results for example when simulating a signal propagates in free space. The finite difference time domain method uses to solve the Maxwell's equations in two-dimensional system. Several cases are simulated such as emit EM radiation from a point source. The transverse magnetic (TM) to z-polarized can be used to simulate such as the propagation of waves in free space or a computational domain contains the obstacles made of a perfect electric conductor.

3. Results and Discussion

We wrote computer programs by using MATLAB (R2013a) to find the solutions for many cases such as electromagnetic waves propagate in free space in two and three dimensions. Equations (4) are written to simulate 3D-FDTD and also the TM wave formulations in equations (7) are written in a computer program to simulate the 2D-FDTD. We constructed the circular, square, cylinder and cube shapes that made of a perfect electric conductor (PECs) in two and three dimensions, respectively as the mesh modes in domains. Therefore, space is divided into small cells and filled each pixel and voxel in 2D and 3D dimensions by the properties of perfect electric conductor and free space at exact locations. The grid resolution is selected at least ten sample points per wavelength and grid dimensions set as 100×100 in two-dimensional model in the *x* and *y* directions and $100 \times 100 \times 100$ in three-dimensional model in the *x*, *y* and *z* directions. The time step is selected to obtain a good stability based on the courant condition for example in 2D-FDTD case can be given as [6]:

$$\Delta t \leq \frac{1}{c\,(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2})^{1/2}} \tag{10}$$

Where c is the speed of light in free space and Δx and Δy are the space increments.

The importance of equation (10) is that it provides a good stability which leads to achieve good results. With regard to a source of excitation that is used to generate electromagnetic fields, there are two types of sources such as a hard source and soft source can be used in FDTD and the simplest type of source is a hard source which was applied in all simulations. The source is setting to electric field to the amount of source[7]. The computational domain is excited with a sinusoidal source with 1.5 GHz frequency. The location of the excitation source for example in two dimensions case is located at a point (50, 50) in middle of a domain as shown in figure 1 and at left and right sides as shown in figure 2 (A, C), respectively. It can be observed that the signals are propagated bilaterally in the positive and negative directions as shown in the snapshot that is taken at 280-time steps as demonstrated in figure 1. When the electromagnetic waves reached the edges, the wave was absorbed, and the images appeared very good. When placing the source at the left side of a computational domain, the left portion of a signal is absorbed while the second portion is propagating in free space at the right. However, the source is placed at right side, the right portion of the signal is absorbed whereas the second portion is propagating at the left as the numerical results are shown in figure 2.

Therefore, the absorbing boundaries conditions (ABCs) must be included in the computations space to limit the spatial domain. The first order Mur's boundary is implemented in 3D-FDTD and second order Mur's boundary is implemented in 2D-FDTD. It can be clearly seen that the TM waves absorbed at these boundaries. We expect that the good results will be achieved as demonstrated in figure 1, the waves at four edges absorbed and appeared extremely good. The radiation patterns of the waves generated as circular patterns as the example shown in figure 1 and figure 2. The results of simulations indicated that the Mur (ABCs) is the supper absorbing boundary conditions. For the example in figure 3, the simulation is reached at a steady state solution. This example demonstrates that good signal is obtained, this means that the signals absorbed at all edges and if there are no ABCs the signals will reflected back in a domain and will affect the distribution of the wave. This can be observed when placing a probe in a domain to record a signal at a specific cell. This will proof that Mur's absorbing boundary is very good for using to simulate many problems for example scattering electromagnetic waves with a PEC structure. Therefore, we can use to solve many cases for the example added obstacle in a space. In this research electromagnetic behaviors are studied as the signals propagate in space and interacted with a PEC as shown in figure 4 as placed a wire in a domain as well as in figure 5 and figure 6 when placing square and circular shapes, respectively. It can be observed from the images that the signals are generated after the obstacle due to the diffractions. Furthermore, by comparing figure 2 and figure 4, it can be noted that the TM wave distributions affected, and the diffraction appeared by adding a wire in space. With regard to three-dimensional, the structures of interest are constructed in the domain in three dimensions and a domain contains the obstacles which are made of perfect electric conductors as the mesh modes which are placed in three-dimensional in the center of a space. Therefore, the cylinder has a diameter equal to 26 cells and length of 30 cells and each voxel in this region set as a perfect electric conductor. Similar approach was utilized with a cube shape and the dimension of each side of cube is 20 cells and set as a PEC. The spatial discretization is chosen to be 0.1 λ in all simulations. Furthermore, the obstacles can be seen in 3D model as a slice in different planes as the example in the x-y plane and x-z plane as demonstrated in figure 7. It was found that electromagnetic wave excited in space and interacted with the obstacles in the domain as shown in figures 8, 9, 10 which can be compared with figure 6 when adding circular shape in two dimensions model. Figs 8, 9 and 10 showed that the EM distributions as the images of electromagnetic fields components in the *x-y*, *x-z* and *y-z* planes, respectively. The similar distributions will be seen when adding the cube as a perfect electric conductor in a domain as shown in figure 11. Figures 12, 13 and 14 are illustrated the distributions of the field components as the images in different planes in the *x-y*, *y-z* and *x-z* planes, respectively. The similar simulations results obtained when simulated 2D and 3D. For example, we obtained the similar distributions, and we can determine the type of the obstacle in the domain as shown in figure 5 and figure 14.

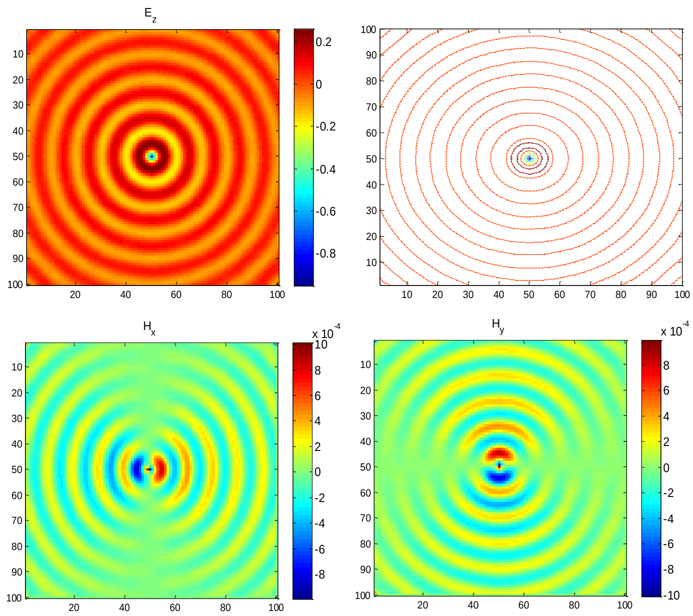

Figure 1 2D-FDTD simulation result: a snapshot of the TM wave distributions generated in the middle of a spatial domain and the waves absorbed at walls.

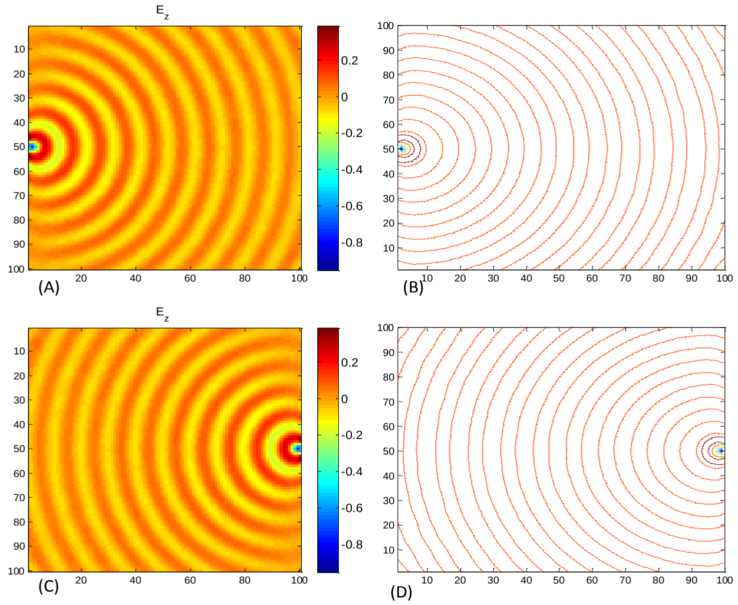

Figure 2 The TM wave generated by the source placed at: (A) the left side at a node (*i*, *j*) = (2, 50) and (C) the right side at a node (98, 50).

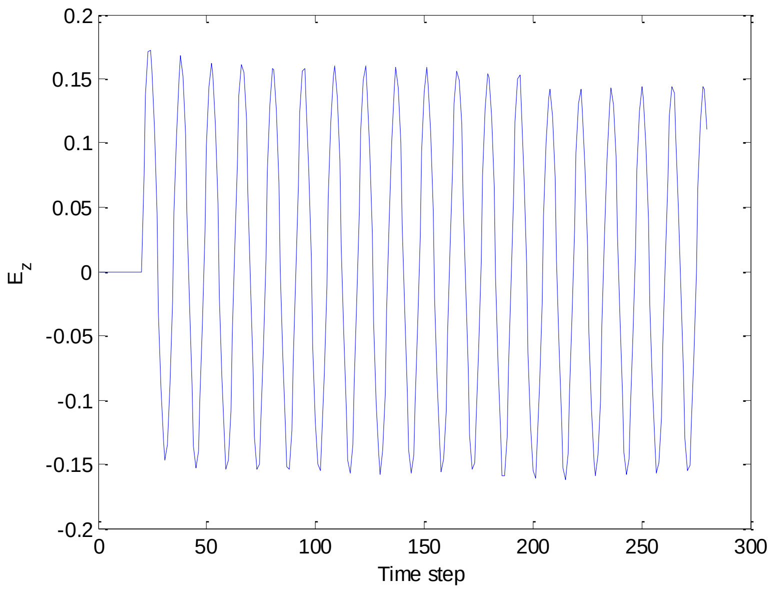

Figure 3 Simulation reached a steady state.

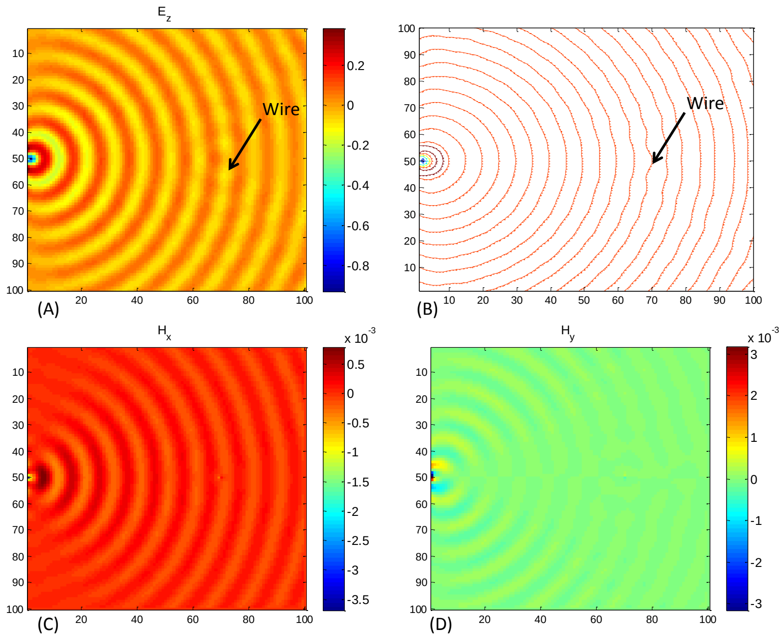

Figure 4 Wire is placed at a domain on the right side and a source on the left side.

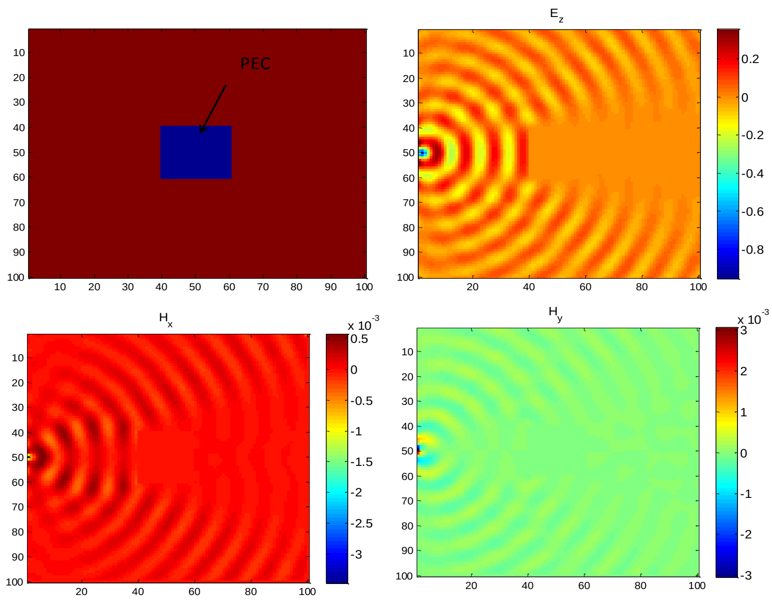

Figure 5 The TM wave distributed in a domain that is included a small square obstacle made of a PEC.

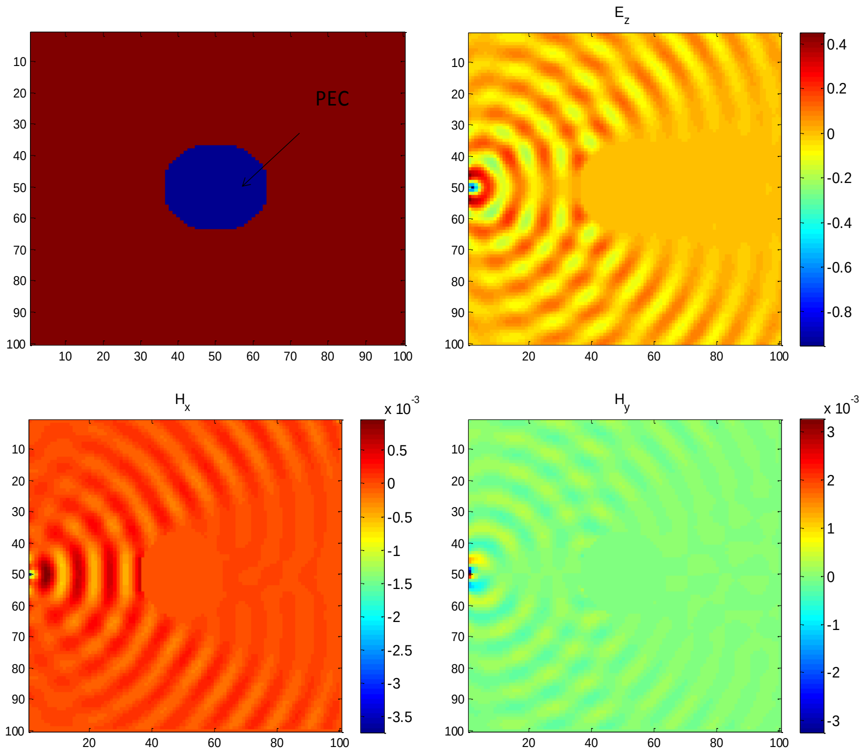

Figure 6 The TM wave distributed in a domain that is included a circular shape made of a PEC in a domain.

Cylinder shape in three dimensions model:

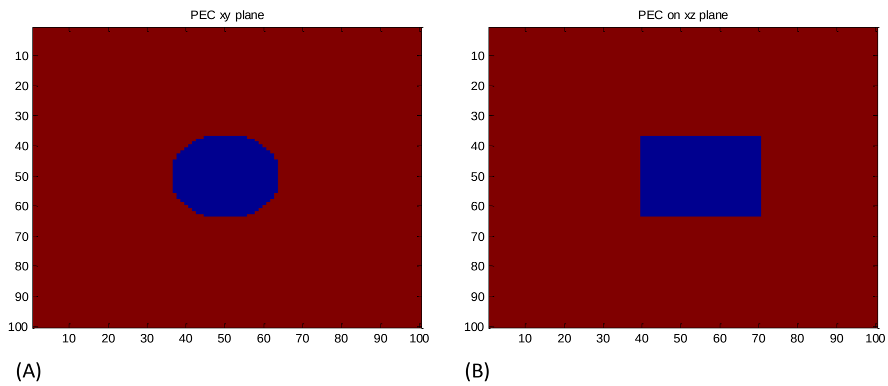

Figure 7 Three dimensions obstacle (cylinder shape) made of a PEC placed in free space.

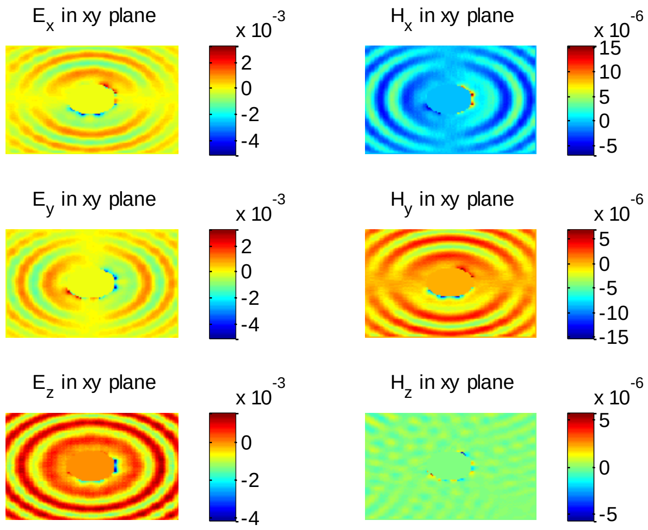

Figure 8 Electromagnetic fields distributions in the *x-y* plane in the presence of the obstacle.

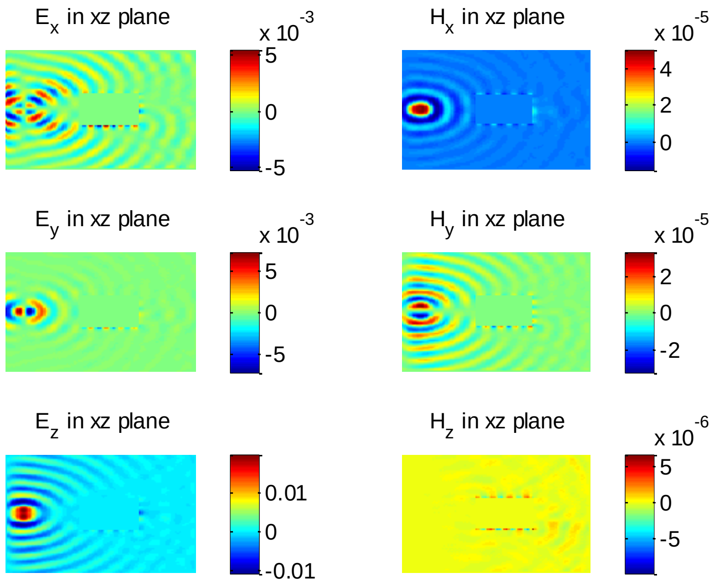

Figure 9 Electromagnetic fields distributions in the x-z plane.

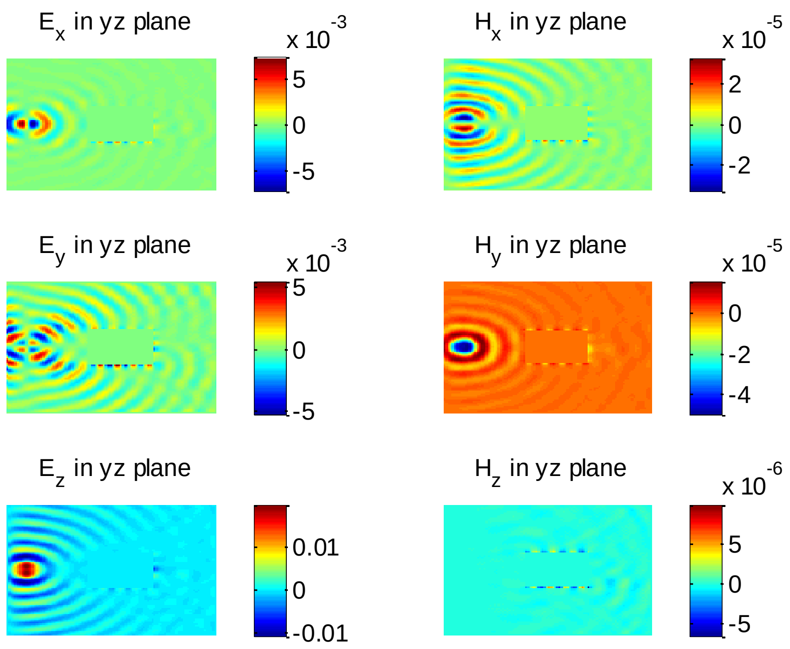

Figure 10 Electromagnetic fields distributions in the y-z plane.

Cube shape in three dimensions model:

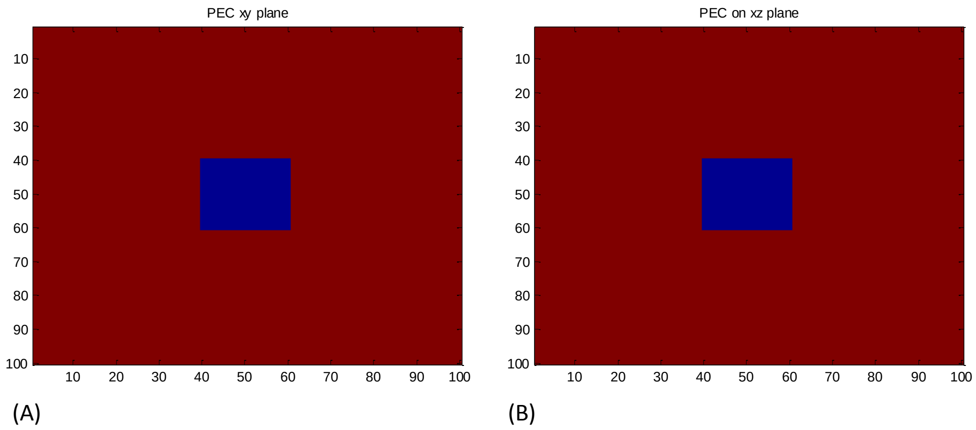

Figure 11 Cube shape made of PEC is placed in a domain in free space.

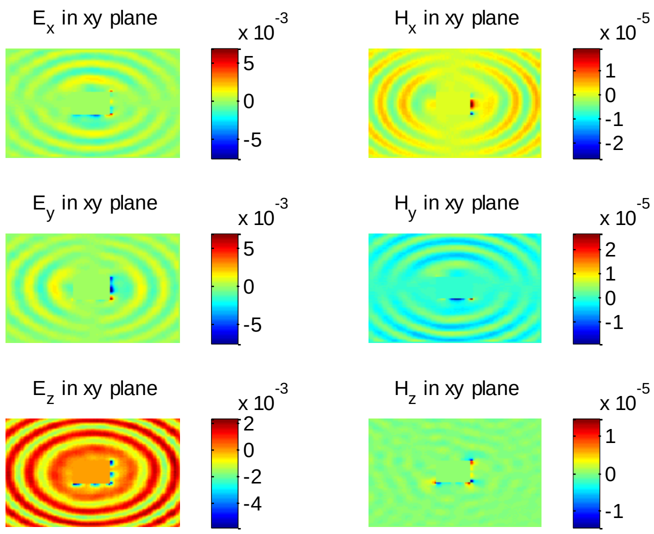

Figure 12 Electromagnetic fields distributions in the *x-y* plane in the presence of the obstacle in a domain.

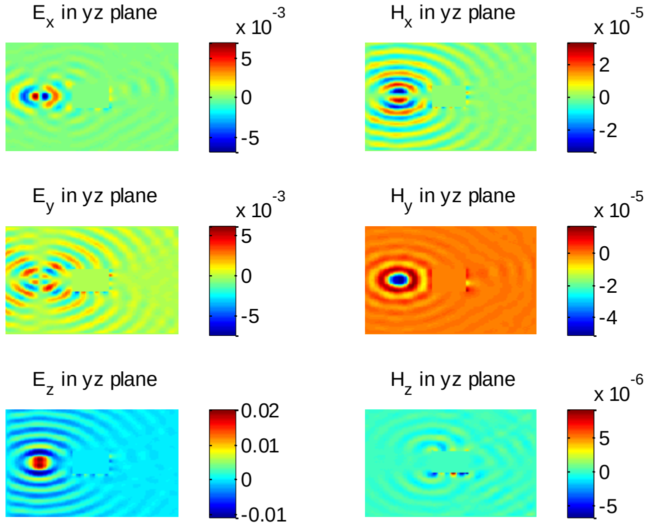

Figure 13 Electromagnetic fields distributions in the y-z plane.

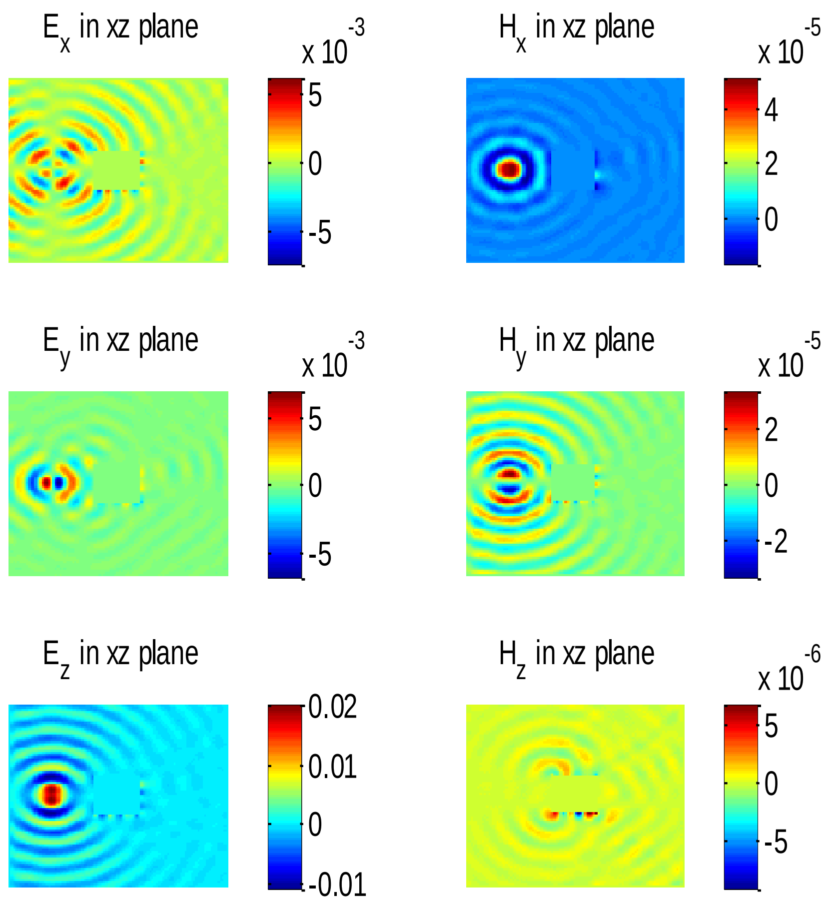

Figure 14 Electromagnetic fields distributions in the *x-z* plane.

4. Conclusion

This research implements the concepts of the FDTD technique as the TM wave in a two-dimensional scheme and described the use of Mur absorbing boundaries conditions (ABCs). As results, Maxwell's equations are solved numerical in the 2-D and 3-D as well as the electromagnetic waves generated and propagated in 3-D and 2-D systems. It was found that Mur ABCs are sufficient boundaries conditions to implement in this study. It can be concluded that the finite difference time domain method is very good technique to find a solution of Maxwell's equations numerically for complicated structure.

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